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Temperature and Flow Fields of a Second Order Fluid Adjacent to a Vertical Stretching Sheet

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ABSTRACT

A steady two-dimensional mixed convection of an incompressible second-order fluid adjacent to a hot vertical stretching sheet which is kept at an uniform wall temperature is studied. A parameter, Gr/Re^2, which is used to represent the dominance of the buoyant effect, is present in governing equations. The local similar transformation, the perturbation expansion and the shooting method are use to analyze the present problem. The numerical solutions of the flow velocity distributions, temperature profiles, the wall shear stress, and the heat transfer of the local similar boundary-layer flow are carried out as functions of the viscoelastic parameter k_1, the Prandtl number Pr, and the buoyancy parameter Gr/Re^2 for the case with the local similar parameter $\xi = 1.0$. The effects of these parameters are also discussed.

Key words: *a second-order fluid, a vertical stretching sheet, mixed convective heat transfer.*

I. Introduction

The analysis of the flow properties of non-Newtonian in a boundary layer are very important in the fields of fluid dynamics and heat transfer. The velocity and temperature distributions within the boundary-layer flow over a continuous moving solid surface also have great importance in engineering applications, for example in the extrusion of a material and heat-treated materials that travel between feed and wind-up rollers or on conveyer belts. The flow entrains the ambient fluid and has different behavior in comparison with the Blasius boundary-layer flow. Flow in the boundary layer over a continuous solid surface with constant speed was first studied by Sakiadis [1]. Erickson *et al* [2]. studied the same flow field and extended the problem to include suction and injection on the moving surface. Their study assumed an inextensible polymer sheet extruded from a die, but McCormack and Crane [3] suggested that a stretching plastic sheet should also be considered in order to reveal the effects due to the extension nature of polymer extrusion. Gupta and Gupta [4] used the similar transformation to study the heat and mass transfer on a stretching sheet with suction and blowing. Danberg and Fansler [5] investigated a steady two-dimensional incompressible boundary-layer flow caused by the stretching of an elastic flat sheet. An uniform stress is applied in their study to produce a linearly varying velocity, with is proportional to the distance between the local point and a fixed reference point, along the surface of the sheet. However, a Newtonian fluid flow model was used in studies mentioned above.

Due to the growing importance of non-Newtonian fluids in technology in recent years, fundamental studies in the boundary layer flow of non-Newtonian fluids have been widely emphasized. The laminar boundary layer flow on an inextensible continuous flat surface moving with a constant velocity in its own plane in a non-Newtonian fluid characterized by a power law model (Oswald-de Wael fluid) was studied by Fox *ed al.* [6] with both exact and approximate methods. However, no elastic effects were presented due to the model used in their study. Siddappa and Khapate [7] extended above problem for a special class of non-Newtonian fluids known as second-order fluids which are viscoelastic in nature to include elastic effects. Siddappa and Hiremath [8] discussed the viscoelastic flow past a stretching plane with suction. Rajagopal *et al.* [9] independently examined the same flow of a second-order fluid as in [7], obtained similarity solutions of the boundary layer equations numerically for the case of small viscoelastic parameter k_1 and shoeed that the skin friction decreases with increase in k_1. Recently, Troy *ed al.* [10] obtained an exact solution to the problem of Rajagopal *et al.* [9], and Chang [11] studied and presented another solution for the same problem. Dandapat and Gupta [12] also examined the above flow with heat transfer, presented an exact analytical solution of the nonlinear equations governing this self-similar flow, which is consistent with the numerical results given in [9], and obtained the solutions for the temperature distributions for various values of k_1. Rajagopal *ed al.* [13] extended their problem in [9] by including an uniform free stream, and indicated that the location of the separation point of the boundary layer depends critically on the nature of the material. Very recently, Vajravelu and Soewono [14] studied the existence, uniqueness and behavior of exact solutions for the combined free and forced convection flow of a second order fluid adjacent to a vertical stretching sheet. The findings of such a physical phenomenon will have definite bearing on the fabric, plastic and polymer industries. Therefore, in the present investigation, a study was undertaken to provide results for the mixed convection flow of a second order fluid adjacent to a hot vertical stretching sheet, held a constant temperature higher than that of the ambient fluid.

The buoyant force is important in the present problem due to the differences between the sheet temperature and the fluid temperature. A complicated flow pattern might occur because of the interaction of the buoyancy and the viscoelasticity of the fluid. Thus, considerable efforts were directed toward the analysis and the understanding of the problem, which is characterized by a setoff highly nonlinear, coupled partial differential equations. The major components of the system include a hot vertical stretching sheet, a mixed convective heat transfer mechanism and a second order fluid. In addition, since non-similarity arisen due to the elastic property of the fluid and the buoyant effect, similarity solutions did not exist. A non-similar derivation technique was used and resulting non-similar equations were solved by using the method of local similarity with a perturbation procedure, which was suggested by Bear and Walter [15]. The effects of the viscoelastic parameter k_1, the buoyancy parameter Gr/Re^2, and the Prandtl number Pr to the local heat transfer and the skin friction along the sheet were discussed in the present study.

II. Theory and Analysis

An incompressible, homogeneous, non-Newtonian, second-order fluid having a constitutive equation based on the postulate of gradually fading memory suggested by Coleman and Noll [16] was used in the present flow. The model equation is expressed as follows:

$$\mathrm{T} = -P\mathrm{I} + \mu \mathrm{A}_1 + \alpha_1 \mathrm{A}_2 + \alpha_2 \mathrm{A}_1^2 \quad (1)$$

where **T** is the stress tensor, P is the pressure, μ is the dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients which are related to the material modulus ($\alpha_1 < 0$ for the present second-order fluid). The kinematic tensors $\mathbf{A_1}$ and $\mathbf{A_2}$ are defined as

$$\mathrm{A}_1 = grad\ \mathrm{V} + (grad\ \mathrm{V})^{\mathrm{T}} \quad (2)$$

$$\mathrm{A2} = \frac{d}{dt}\mathrm{A1} + \mathrm{A1} \cdot (grad\ \mathrm{V}) + (grad\ \mathrm{V})\mathrm{T} \cdot \mathrm{A1} \quad (3)$$

Where **V** are velocities and $\frac{d}{dt}$ is the material time derivative. As mentioned in Markovitz and Coleman [17] and Acrivos [18]. This model is applicable to some dilute polymers.

In the present analysis we consider the flow of a second-order fluid obeying equation (1) adjacent to a hot vertical stretching sheet (or wall) coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the *x*-axis (the positive *x*-axis is taken vertically upward and parallel to the direction of gravity) so that the wall is stretched keeping the origin fixed, and kept a velocity proportional to *x* (the proportionality constant is termed "stretching rate"). The geometric model is shown in Fig. 1. The boundary-layer flow, with zero free stream

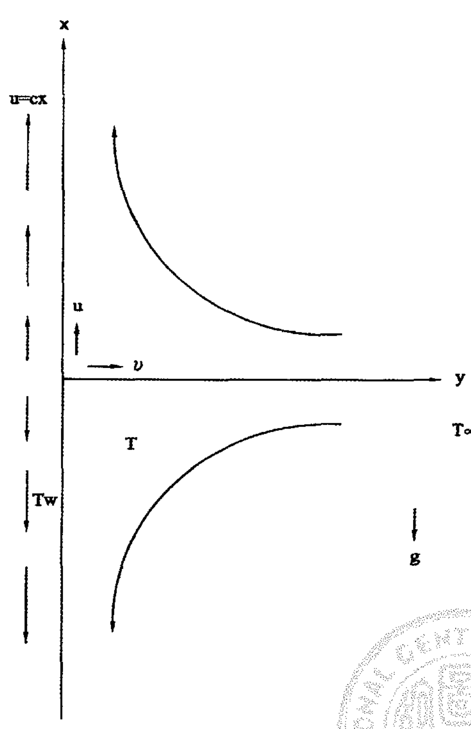

Fig.1 Physical model

velocity, is due to the stretching of the sheet and the buoyancy effect.

The steady two-dimensional boundary-layer equations for this flow and heat transfer, in usual notation, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k[\frac{\partial}{\partial x}(u\frac{\partial^2 u}{\partial y^2})+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}] + g_x\beta(T-T_\infty) \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{6}$$

where u, v are the velocity components in the x and y direction, T the fluid temperature adjacent to the sheet, g_x the magnitude of the gravity, α the thermal diffusivity, β the coefficient of thermal expansion, ν the kinematic viscosity, and $k=-\alpha_1/\rho$ the elastic parameter. The well-know Boussinesq approximation is used to represent the buoyancy force term. The boundary conditions to the problem are

$$u=cx(c>0),\quad v=0,\quad T=T_w \quad at\ y=0 \tag{7a}$$

$$u\to 0,\quad \frac{\partial u}{\partial y}\to 0,\quad T\to T_\infty \quad at\ y\to\infty \tag{7b}$$

where T_w and T_∞ are constant wall temperature and ambient fluid temperature, respectively. In the present study, we focus primary attention on results for $T_w > T_\infty$ and $x > 0$.

Using the stream function φ, such that

$$u=\frac{\partial\varphi}{\partial y},\quad v=-\frac{\partial\varphi}{\partial x} \tag{8}$$

togethwr with the transformations

$$\xi=\frac{x}{L},\quad \eta=(\frac{c}{\nu})^{\frac{1}{2}}y \tag{9}$$

$$\varphi=(\nu c)^{\frac{1}{2}}xf(\xi,\eta) \tag{10}$$

$$\theta(\xi,\eta)=(T-T_\infty)/(T_w-T_\infty) \tag{11}$$

and in terms of these new variables, we obtain the velocity components as

$$u=cxf'(\xi,\eta),$$

$$v=-(\nu c)^{\frac{1}{2}}[f(\xi,\eta)+\xi\frac{\partial f(\xi,\eta)}{\partial\xi}] \tag{12}$$

and the governing equations (5) and (6) as

$$(f')^2-ff''+\xi(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi})=$$

$$f'''-k_1[2f'f'''-(f'')^2-ff''''+\xi(f'''\frac{\partial f'}{\partial\xi}+f'\frac{\partial f'''}{\partial\xi}$$

$$-f''\frac{\partial f''}{\partial\xi}-f''''\frac{\partial f}{\partial\xi})]+\xi^{-1}\frac{Gr}{\mathrm{Re}^2}\theta \tag{13}$$

$$\theta''+\Pr f\theta'+\Pr\xi(\theta'\frac{\partial f}{\partial\xi}-f'\frac{\partial\theta}{\partial\xi})=0 \tag{14}$$

where the primes in equations are denoted as partial differentiation with respect to η. The boundary conditions (7a) and (7b) for equations are defined as

$$f'(\xi,0)=1.0,\quad f(\xi,0)+\xi\frac{\partial f(\xi,0)}{\partial\xi}=0,$$

$$\theta(\xi,0)=1.0 \quad at\ \eta=0 \tag{15a}$$

$$f'(\xi,\infty)\to 0,\quad f''(\xi,0)\to 0,\quad \theta(\xi,\infty)\to 0$$

$$as\ \eta\to\infty \tag{15b}$$

The parameters k_1, Pr, Gr and Re in the equations are defined as

$$k_1=kc/\nu,\quad Pr=\nu/\alpha,$$

$$Gr=g_x\beta(T_w-T_\infty)L^3/\nu^2,\quad Re=UL/\nu \tag{16}$$

where L is the characteristic length of the sheet and $U=cL$ is chosen as the character istic velocity of the stretch, k_1, Pr, Gr and Re are the viscoelastic parameter, the Prandtl number, the Grashof number and the Reynolds number, respectively. These parameters control variations of the flow and the heat transfer characteristics.

In the present study, isothermal condition is considered along the sheet, and it does not admit similar solutions. It should be noted that, as a special case, Vajravelu and

Soewono [14] used $T_w = T_\infty + Ax/L$, a linearly varying temperature distribution as the wall temperature, obtained similar boundary layer equations and studied the uniqueness and extstence of solutions for the mixed convection flow of a second-order fluid adjacent to a vertical stretching sheet. Among the presently available approaches for treating such problems, the local similarity method Kays [19], is perhaps the one of the most frequently employed, owing to its conceptual and computational simplicity. One of the especially attractive features of the local similarity method is that the solution at a particular streamwise location can be found without having to perform calculations at upstream locations, that is, each solution is locally autonomous. Anther advantage of the method is that the governing equations encountered in the course of its application can be treated as ordinary differential equations and resemble those equation for similar boundary layer flows. In practice, according to this approach, the quantity ξ may be regarded as a constant parameter at any streamwise location, and terms containing partial derivatives with respect to ξ are postulated to be sufficiently small so that they may be approximated by zero. This gives

$$(f')^2 - ff'' = f''' - k_1[2f'f''' - (f'')^2 - ff''''] + \xi^{-1}\frac{Gr}{\mathrm{Re}^2}\theta \quad (17)$$

$$\theta'' + \Pr f\theta' = 0 \quad (18)$$

$$f(\xi,0) = 0, \quad f'(\xi,0) = 1.0, \quad \theta(\xi,0) = 1.0, \quad at \ \eta = 0 \quad (19a)$$

$$f'(\xi,\infty) \to 0, \quad f''(\xi,0) \to 0, \quad \theta(\xi,\infty) \to 0, \quad \text{as} \ \eta \to \infty \quad (19b)$$

Thus, although equations (17)-(19a,b) are partial differential equations, they can be treated as a set of coupled ordinary differential equations of the similarity type and may be solved by the usual techniques for similar boundary layer flows. As mentioned before, the solution corresponding to any given ζ value is independent of the solution at any other ζ. These equations become uncoupled when $Gr/Re^2 = 0$, and the flow is regarded as a pure forced convective flow.

III. Numerical Technique

We assume that a set of local similar solutions of equations (17) and (18) can be expanded as power series in k_1. Following Bear and Walters [15], and assuming a small k_1, local similar solution are written as

$$f(\xi,\eta) = f_0(\xi,\eta) + k_1 f_1(\xi,\eta) + k_1^2 f_2(\xi,\eta) + \cdots\cdots \quad (20)$$

$$\theta(\xi,\eta) = \theta_0(\xi,\eta) + k_1\theta_1(\xi,\eta) + k_1^2\theta_2(\xi,\eta) + \cdots\cdots \quad (21)$$

Substituting equations (20) and (21) into equation (17)-(19a,b) and equating coefficients of k_1^0 and k_1^1 (because k_1 is small, computing equations up to first-order of k_1 is adequate) results in

Order k_1^0:

$$(f_0')^2 - f_0 f_0'' = f_0''' + \xi^{-1}\frac{Gr}{\mathrm{Re}^2}\theta_0 \quad (22)$$

$$\theta_0'' + \Pr f_0\theta_0' = 0 \quad (23)$$

$$f_0 = 0, \quad f_0' = 1.0, \quad \theta_0 = 1.0, \ at \ \eta = 0 \quad (24a)$$

$$f_0' \to 0, \quad f_0'' \to 0, \quad \theta_0 \to 0, \ \text{as} \ \eta \to \infty \quad (24b)$$

Order k_1^0:

$$f_1''' + f_1 f_0'' + f_0' f_1'' - 2f_0' f_0' + \xi^{-1}\frac{Gr}{\mathrm{Re}^2}\theta_1 = 2f_0' f_0''' - (f_0'')^2 - f_0 f_0'''' \quad (25)$$

$$\theta_1'' + \Pr(f_0\theta_1' + f_1\theta_0') = 0 \quad (26)$$

$$f_1 = 0, \quad f_1' = 0, \quad \theta_1 = 0, \quad at \ \eta = 0 \quad (27a)$$

$$f_1' \to 0, \quad f_1'' \to 0, \quad \theta_1 \to 0, \quad \text{as} \ \eta \to \infty \quad (27b)$$

The local similar solutions of equations (22)-(27a,b) can be obtained by the fourth-order Runge-Kutta integration with a systematic guessing of missing boundary condition at $\eta = 0$ (served as initial conditions of the numerical integration), which lead the integration to satisfy the boundary conditions specified at large η. This numerical algorithm is termed the shooting method.

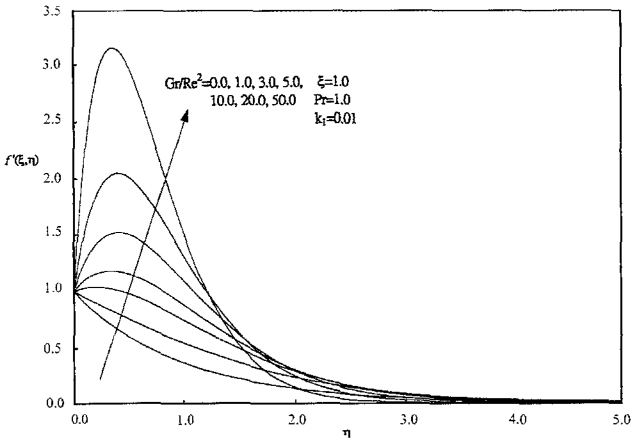

Fig.2 $f'(\xi,\eta)$ vs. η with ξ = 1.0, Pr = 1.0, k_1 = 0.01 and various values of Gr/Re^2.

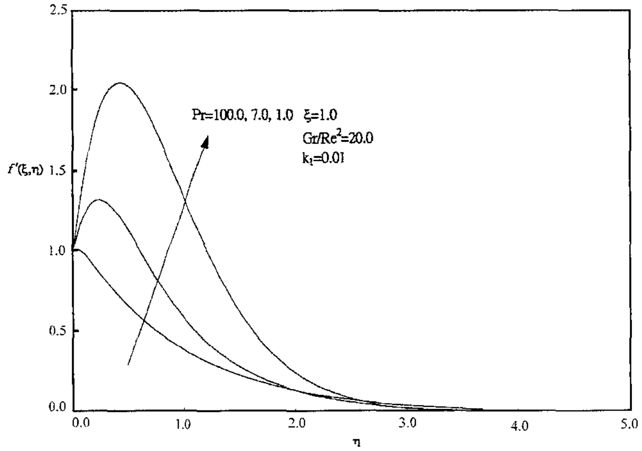

Fig.4 $f'(\xi,\eta)$ vs. η with ξ = 1.0. Gr/Re^2 = 20, $k1$ = 0.01 and various values of. Pr.

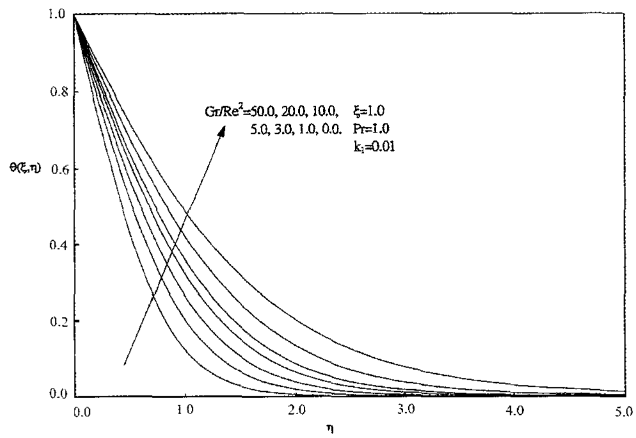

Fig.3 $\theta(\xi,\eta)$ vs. η with ξ = 1.0, Pr = 1.0, k_1 = 0.01 and various values of Gr/Re^2 .

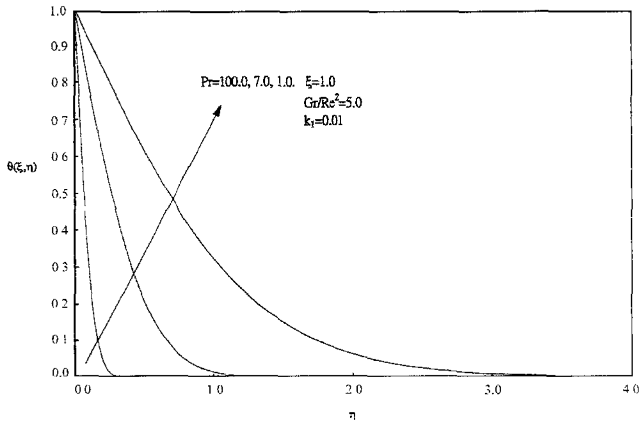

Fig.5 $\theta(\xi,\eta)$ vs. η with ξ = 1.0. Gr/Re^2 = 20, k_1 = 0.01 and various values of. Pr.

IV. Results and Discussion

The shooting method and fourth order Runge-Kutta integration were used to solve the zero-order equations and first-order perturbed equation for the velocity and temperature fields. The truncation error of the integration is O($\Delta\eta^4$), according to Burden and Faires [20], and the largest grid size $\Delta\eta$ is 0.01. The numerical computations were carried out for the values of controlling parameters Gr/Re^2 ranging from 0 to 50.0, Pr ranging from 1.0 to 100.0, and k_1 ranging from 0.005 to 0.09. The local similar variable ξ was chosen as 1.0 for simplicity. In fact, the coefficient of buoyancy terms in equations (22) and (25) is ξ^{-1} Gr/Re^2, a combination of ξ variable and buoyant parameter Gr/Re^2. Therefore, it is reasonable to pick $\xi = 10$ in the present study.

The streamwise velocity $u(x,y)$ is represented by the function $f'(\xi,\eta)$ as shown in equation (12). Variation of profiles of $f'(\xi,\eta)$ versus η for various values of the parameter Gr/Re^2 with $\xi = 1.0, \Pr = 1.0$, and k_1 = 0.01 is shown in Fig. 2. These curves strikingly display the role of the free convection in the flow field. As Gr/Re^2 ranging from 0. to 50.0 these profiles indicated that the mode of heat transfer changes from pure forced convection dominance to free convection domainance. In the boundary layer, the buoyant force, which is larger with larger Gr/Re^2, accelerates the flow, carries heat away fastly, and in term enhances the heat transfer performance. The effect is also present in corresponding distributions of the temperature function

$\theta(\xi,\eta)$, which indicates that a larger temperature gradient is present in the flow, at higher value of the parameter Gr/Re^2. A larger heat transfer coefficient is obtained due to a larger temperature gradient. Consequently, a better heat transfer is resulted.

The influence of the Prandtl number on the profile of $f'(\xi,\eta)$ is depicted in Fig.4 for $Gr/Re^2 = 20$ and $k_1 = 0.01$ at $\xi = 1.0$. It is shown that the buoyant effect on the distribution of $f'(\xi,\eta)$ is less significant with a higher Prandtl number. The varation of $\theta(\xi,\eta)$ versus η for various of Prandtl number with $Gr/Re^2 = 20$ and $k_1 = 0.01$ at $\xi = 1.0$. is shown in Fig. 5. The gradient of dimensionless temperature $\theta(\xi,\eta)$ increases with an increase in Prandtl number. This is consistent with a well-known result that the thermal boundary layer thickness for Newtonian fluid flow decreases with increasing Pr. Therefore, with a fixed value of k_1, the effects of the prandtl number and buoyancy on velocity and temperature profiles of a second-order fluid are similar to a Newtonian fluid.

The results of greatest practical interest are the effects of the mixed convection parameter Gr/Re^2, the viscoelastic parameter k_1 and the Pranddtl number Pr on the skin friction and heat transfer characteristics of the problem. The skin friction along the stretching sheet is related to the wall shear stress. The wall shear stress is obtain by using equations (1), (2), (3) and (7a) and is written as

$$T_{xy}\,|_{y=0} = \mu\frac{\partial u}{\partial y} + \alpha_1\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + u\frac{\partial^2 u}{\partial x\partial y}\right)|_{y=0} \quad (28)$$

The dimensionless from of the wall shear stress becomes

$$\tau_w = \rho^{-1}x^{-1}c^{-3/2}\nu^{-1/2}(T_{xy})_{y=0} = (1-3k_1)f''(\xi,0) \quad (29)$$

The heat transfer coefficient h_c along the stretching sheet is related to the Nusselt number Nu

$$Nu = \frac{h_c L}{K_c} = -\frac{(\frac{\partial T}{\partial y})_{y=0}}{(T_w - T_\infty)/L} \quad (30)$$

now reads

$$Nu\,\mathrm{Re}^{-\frac{1}{2}} = -\theta'(\xi,0) \quad (31)$$

where k_c is the conductivity of the fluid. The dimensionless wall shear stress and the dimensionless heat transfer coefficient as calculated from the partial sums of the series equations (20) and (21), namely,

$$\tau_w = (1-3k_1)[f_0''(\xi,0) + k_1 f_1''(\xi,0)] \quad (32)$$

and

$$Nu\,\mathrm{Re}^{-\frac{1}{2}} = -[\theta_0'(\xi,0) + k_1\theta_1'(\xi,0)] \quad (33)$$

in the present work are respectively given in Table 1 and Table 2. For small Gr/Re^2 the first-order term $k_1 f_1''(\xi,0)$ of equation (32) is small relative to the zeroth-order term $f_0''(\xi,0)$, and both $f_0''(\xi,0)$ and $f_1''(\xi,0)$ are negative. Table 1 shows that the magnitude of τ_w decreases with increasing values of the parameter k_1 and increases with increasing values of the Prandtl number Pr when forced convection effect is predominant (i.e. at lower values of Gr/Re^2), and is independent of the Prandtl number for the limiting case of the pure forced convection (i.e. Gr/Re^2 is further increased, $k_1 f_1''(\xi,0)$ gains importance, and both $f_0''(\xi,0)$ and $f_1''(\xi,0)$ become positive. Hence, at fixed Gr/Re^2, the magnitude of τ_w depends on the relative quantities of $f_0''(\xi,0)$, $f_1''(\xi,0)$ and k_1, and there is no general rule for the variation of the magnitude of τ_w. In addition, with moderate Gr/Re^2 and fixed values of k_1, τ_w changes sign from negative to positive when Pr varies from 100.0 to 1.0. However, with larger Gr/Re^2, values of τ_w are positive even Pr is larger. Values of $-\theta'(\xi,0)$ are listed in Table2 according to variations of Pr, k_1 and Gr/Re^2. $-\theta'(\xi,0)$ increases with increasing values of Pr and/or Gr/Re^2. It is also shown that $-\theta'(\xi,0)$ decreases whit increasing k_1 at lower values of Gr/Re^2 (forced convection predominant). Similar results were obtained by Dandapat and Gupta [12] and Cortell [21] for the pure forced convection. However, $-\theta'(\xi,0)$ increases with increasing k_1 at larger values of Gr/Re^2 (free convection predominant).

V. Conclusion

A steady two-dimensional mixed convection of an incompressible second-order fluid adjacent to a hot vertical stretching sheet is studied. Local similar solutions were obtained and results indicate that the buoyant force can accelerate the fluid speed in the boundary layer and enhance the heat transfer performance. The variation of the magnitude of dimensionless wall shear stress τ_w depends on relative quantities of k_1, Gr/Re^2, $f_0''(\xi,0)$ and $f_1''(\xi,0)$.

Table 1. Dimensionless shear stress τ_w , at the wall with $\xi = 1.0$

Pr	K_1	Gr/Re^2=0.0	1.0	3.0	5.0	10.0	20.0	50.0
	0.005	-0.987469	-0.485168	0.345154	1.082016	2.728228	5.606242	12.893267
	0.01	-0.974862	-0.477484	0.347272	1.081221	2.726403	5.617388	13.002538
1. 0	0.03	-0.923684	-0.446838	0.353463	1.073189	2.706814	5.632118	13.345101
	0.05	-0.871303	-0.416339	0.356005	1.057397	2.667570	5.599086	13.536433
	0.09	-0.762932	-0.355780	0.350142	1.002532	2.530115	5.389736	13.465400
	0.005	-0.987469	-0.739995	-0.270306	0.174470	1.212747	3.095331	7.992378
	0.01	-0.974862	-0.728917	-0.261745	0.181094	1.216413	3.098597	8.019295
7. 0	0.03	-0.923684	-0.684544	-0.228875	0.204718	1.224230	3.096073	8.080996
	0.05	-0.871303	-0.640078	-0.198205	0.223749	1.221095	3.068607	8.069148
	0.09	-0.762932	-0.550862	-0.143464	0.248035	1.181966	2.938849	7.824809
	0.005	-0.987469	-0.912126	-0.762095	-0.612913	-0.243500	0.481349	2.563493
	0.01	-0.974862	-0.899681	-0.749960	-0.601072	-0.232327	0.491467	2.572465
100. 0	0.03	-0.923684	-0.849453	-0.701584	-0.554485	-0.189944	0.526544	2.593503
	0.05	-0.871303	-0.798509	-0.653470	-0.509139	-0.151255	0.552987	2.590784
	0.09	-0.762932	-0.694478	-0.558028	-0.422167	-0.084958	0.579970	2.514073

Table 2 . Values of $-\theta'(\xi,0)$, at the wall with $\xi = 1.0$

Pr	K_1	Gr/Re^2=0.0	1.0	3.0	5.0	10.0	20.0	50.0
	0.005	0.581456	0.664313	0.748199	0.803497	0.898353	1.019540	1.227934
	0.1	0.580935	0.664258	0.748656	0.804365	0.900094	1.022751	1.234756
1. 0	0.03	0.578852	0.664039	0.750485	0.807839	0.907061	1.035599	1.262041
	0.05	0.576769	0.663819	0.752314	0.811312	0.914027	1.048445	1.289327
	0.09	0.572604	0.663381	0.755972	0.818258	0.927960	1.074139	1.343898
	0.005	1.894851	1.930143	1.991083	2.043197	2.149983	2.309669	2.621406
	0.01	1.894298	1.929960	1.991542	2.044220	2.152229	2.313969	2.630702
7. 0	0.03	1.892088	1.929230	1.993381	2.048312	2.161209	2.331171	2.667887
	0.05	1.889877	1.928499	1.995219	2.052403	2.170189	2.348372	2.705073
	0.09	1.885457	1.927038	1.998896	2.060586	2.188149	2.382775	2.779443
	0.005	7.765088	7.774979	7.794544	7.813828	7.860874	7.950446	8.190397
	0.01	7.764524	7.774574	7.794454	7.814048	7.861850	7.952868	8.196751
100. 0	0.03	7.762268	7.772954	7.794093	7.814927	7.865757	7.962555	8.222162
	0.05	7.760012	7.771334	7.793731	7.815805	7.869663	7.972243	8.247573
	0.09	7.755499	7.768095	7.793008	7.817563	7.877475	7.991618	8.298396

Dimensionless heat transfer coefficient $-\theta'(\xi,0)$ increases with increasing values of Pr and/or Gr/Re^2. $-\theta'(\xi,0)$ also decreases with increasing k_1 at lower values of Gr/Re^2 (forced convection predominant), and increases with increasing k_1 at larger values of Gr/Re^2 (free convection predominant) .

Nomenclature

A : constant.

A_1, A_2 : kinematic tensors.

c : stretching rate.

Gr : Grashof number, $Gr = g_x \beta (T_w - T_\infty) L^3 / \nu^2$.

g_x : gravitational acceleration in the x-direction.

h_c : convective heat transfer coefficient.

L : characteristic length.

k : elastic parameter, $k = -\alpha_1 / \rho$.

k_1 : viscoelastic parameter, $k_1 = kc/\nu$.

k_c = conductivity of the fluid.

Nu : Nusselt number, $Nu = h_c L / k_c$.

P : pressure.

Pr : Prandtl number.

Re : Reynolds number, Re=UL/ν .

T : stress tensor.

T : temperature

T_w : constant wall temperature.

T_∞ : constant ambient temperature.

U : characteristic velocity.

u, v : velocity components in the x and y directions, respectively.

V : velocity vector.

x : vertical coordinate.

y : horizontal coordinate.

α : thermal diffusivity.

$\alpha_{1,2}$: first and second normal stress coefficients.

β : thermal expansion coefficient.

ν : kinematic viscosity.

θ : dimensionless temperature, $\theta = (T - T_\infty) / (T_w - T_\infty)$.

ρ : density of the fluid.

φ : stream function.

τ_w : dimensionless wall shear stress.

ξ : local paramarameteeter, $\xi = x/L$.

η : similar prameter, $\eta = (c/\nu)^{\frac{1}{2}} y$

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二次流體在垂直延伸薄板側之熱流場分析

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摘　要

本文主要探討不可壓縮二次流體，流經一具有均勻壁面溫度的垂直延伸平板，其二維的混合熱對流情形。Gr/Re2 在本文控制方程式中為一個參數，代表浮力主導的效應。本文採用區域相似轉換法、擾動展開法、射值法來分析問題。本文把流場的速度、溫度、壁面剪應力和區域相似邊界層熱傳遞之散佈情形，在相似係數為$\xi=1.0$下，利用數值方法將其表示為黏彈性係數 k1，普朗特數 Pr，浮力參數 Gr/Re2 的函數。這些參數的效應，也在本文中論述。

關鍵詞：二次流體，垂直延伸平板，混合熱對流傳遞。