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Welfare, Output Allocation and Price Discrimination in Input Markets

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ABSTRACT

We analyze the price decision of an upstream monopolist who produces and sells an intermediate good to downstream markets where a backward-integrated chain store competes with many local retailers. It is shown that the upstream monopolist may charge a more efficient local retailer a lower input price under price discrimination. This finding is of interest as it goes against the standard conclusion in the input price discrimination literature. Moreover, contrary to the general findings in the literature in which input price discrimination is welfare-deteriorating, we find that input price discrimination improves social welfare if the positive output allocation efficiency effect outweighs the negative production efficiency effect. These results are robust even if the chain store is not backward integrated.

1. INTRODUCTION

Investigation of the effects on market outputs and social welfare of price discrimination has a long history. Employing the criterion of "adjusted concavity", Robinson (1933) first examines the effect of price discrimination on outputs in two submarkets. The property of the demand curve is found to be an important determinant of the change in total output. For instance, with linear demand curves, the total output is not affected by price discrimination. Schmalensee (1981) extends her model to explore the relationship between the changes in market output and social welfare and shows that price discrimination is welfare-enhancing only if it can increase the total output. Varian (1985) examines the same question and concludes that with linear demand curves, price discrimination necessarily lowers social welfare. The economic explanation for this conclusion is that price discrimination does not change the total output, but it distorts the willingness to pay of the consumers for the last unit of the output in each market, thereby resulting in a lower welfare level. Holahan (1975) employs a linear-market model to examine the same question and finds that, when the market area is endogenous, a monopolist serves more markets (consumers) under price discrimination, which is therefore welfare-improving. Different from Holahan's framework, Hwang and Mai (1990) propose a barbell model and find that if the location of the monopolist is endogenously determined, price discrimination although lowers the total output, it is socially desirable.[1]

Over the last few years, effort has been devoted to the study of price discrimination in input markets. Katz (1987) examines the welfare effect of price discrimination in an intermediate good market where a monopolist sells an intermediate good to many local firms and a chain store. Each local firm serves only its local market and the chain store sets up many branches in order to serve all the downstream markets. In his model, the downstream firms differ in their abilities to integrate backward into the intermediate good. With this framework, he presents the condition under which price discrimination decreases the total output in the final good market and thus reduces social welfare. On the other hand, he also points out that price discrimination is welfare-improving only if inefficient backward integration is prohibited. Moreover, considering the downstream

[1] Hwang and Mai (1990) assume markets exist only at the two ends of the linear market and the plant location is endogenously determined by the monopolist.

R&D investments, DeGraba (1990) finds that discriminatory pricing from an upstream monopolist will discourage downstream firms' efforts in R&D activities and is unfavorable to social welfare. Yoshida (2000) adopts a generalized model to show that an increase in the total output of the final good is a sufficient condition for deterioration in welfare as price discrimination reinforces the market (product) inefficiency of the downstream production.[2] Recently, by assuming that there is an input monopolist facing a threat of demand-side substitution, Inderst and Valletti (2009) find that the more efficient downstream firm always receives a price discount from the upstream monopolist and price discrimination is adverse to social welfare.[3] However, when downstream firms can engage in R&D investments to lower their production costs in the long run, social welfare can thus be enhanced under some reasonable circumstances.

In sum, the literature on input price discrimination has indicated that discriminatory pricing tends to reduce social welfare. However, no one has shown that input price discrimination may improve output allocation among the final good markets and the resulting social welfare. To fill the gap, this paper establishes a theoretical model to encompass both the production efficiency effect and the output allocation effect from input price discrimination. We find that price discrimination is welfare-enhancing if the positive output allociation effect outweighs the negative production efficiency effect.

Following Katz (1987), we assume that there are n independent downstream markets. In each market, there is a branch set by the chain store competing with a local firm (a retailer) who serves its local market only. There is an upstream monopolist producing and selling an intermediate good (or an input) to the local firms in each market.[4] Our model differs from that in Katz (1987) in two aspects. First, instead of assuming the downstream firms have the same marginal cost, we allow their marginal costs to be different. This cost heterogeneity can arise from either the firm-specific and/or the market-specific heterogeneity. The former reflects retailers and the chain store have different marginal costs to produce and/or market their products. The latter reflects different transport costs to ship the intermediate good from the upstream monopolist to each retailer. Second, in Katz (1987), the decision of backward integration

[2] Using a general demand function, Valletti (2003) extends the model of Yoshida (2000) and proves that, under some reasonable assumptions, input price discrimination is adverse to social welfare.

[3] Tyagi (2001) reaches similar conclusions in a model with downstream firms producing differentiated products and interacting repeatedly over time.

[4] For expositional convenience, we shall use "intermediate good" and "input" interchangeably throughout the paper.

by the chain store is endogenously determined and whether it is integrated or not is crucial to the welfare effect of price discrimination. As the focus of the present study is to show how the two effects from price discrimination—output allocation and production efficiency—interact between each other to affect social welfare, the integration decision is assumed to be exogenously given in this paper. We first consider a basic model in which the chain store is backward integrated. The basic model can help us clarify the output allocation effect from price discrimination. After that, we will also consider the situation where the chain store is not backward integrated and purchases the intermediate good from the upstream monopolist.

The main findings of this paper are as follows. First, we show that the equilibrium input price under price discrimination is not always negatively related to production costs of input buyers. Namely, an input buyer with a lower production cost may not be charged a higher input price by the upstream monopolist. In the basic model, we find that a more efficient local firm if competing with a relatively efficient rival (a branch of the chain) shall be charged a lower input price under discriminatory pricing than under uniform pricing. Second, the output allocation efficiency from price discrimination is enhanced in our model if and only if the distribution of market output becomes more even. Finally, even though the total output remains unchanged and the average production cost increases after price discrimination, price discrimination may still enhance social welfare. This result is contrary to the conclusion found in the relevant literature that input price discrimination is welfare-deteriorating. The remainder of this paper is organized as follows. Section 2 introduces the basic model. The optimal prices of an input monopolist under uniform and discriminatory pricing are derived and compared in Section 3. In Section 4, we compare the output levels, output allocation efficiency, production efficiency and social welfare under the two pricing regimes. Section 5 modifies our basic model to consider the cases in which the chain store is allowed to operate in the intermediate good markets. Section 6 concludes this paper.

2. THE BASIC MODEL

We assume there are n separate final good markets with the same demand function as follows:

$$P_i = a - bQ_i, \qquad i = 1, \cdots, n;\ n \geq 2,$$

where $a > 0$, $b > 0$, P_i is the price and Q_i is the output of the i^{th} market. In each market, there are two sellers, a local firm (or a retailer) which serves the respective local market only, and the other is a branch set by a chain store that serves the n markets. The total output in market i is then given as: $Q_i = q_i^c + q_i^\ell$, where the superscripts "c" and "ℓ" represent that the variables are associated with the chain store (or its branch at market i) and a local firm, respectively. In the upstream market, an input monopolist produces and sells an intermediate good to local firms. In order to produce one unit of the final good, each final good producer needs one unit of the intermediate good. The marginal cost of the chain store serving market i is assumed to be v_i. This marginal cost differs in each local market due to, for example, different transport costs of shipping the intermediate good from the chain's plant to the local markets. We further assume that the marginal cost of the final good for the local firm in market i consists of the intermediate good price w_i and a constant manufacturing cost (or marketing cost) θ_i. According to the firm-specific and the market-specific heterogeneity mentioned previously, we assume the cost structure of downstream firms as follows: $v_i \neq v_j$ and $\theta_i \neq \theta_j, \forall i, j \in \{1, 2, \cdots, n\}$ and $v_i \neq \theta_i, \forall i \in \{1, 2, \cdots, n\}$. Let $\overline{\theta} = \left(\sum_{i=1}^{n} \theta_i\right)/n$ and $\overline{v} = \left(\sum_{i=1}^{n} v_i\right)/n$ denote respectively the means of v_i and θ_i. In addition, denote the variances of the distributions of v_i and θ_i and their covariance as σ_v^2, σ_θ^2 and $\sigma_{v\theta}$, respectively. Finally, the marginal cost of the intermediate good is assumed to be zero for both the upstream monopolist and the backward-integrated chain store.[5]

In the present paper, the chain store can be either backward integrated or not. We shall first consider the integrated case in which the chain store produces and supplies the intermediate good only to its branches.[6] In this benchmark model, the welfare of input price discrimination depends on two effects: the production efficiency effect and the output allocation efficiency effect. Furthermore, the model is modified in two directions. First, we assume the branches of the chain store have to purchase the intermediate good from the upstream monopolist. Second, we consider the competition between the incumbent input supplier and the integrated chain store in the intermediate good markets.

[5] The assumption of zero marginal cost for the intermediate good is not crucial for our conclusions.

[6] In our basic model, as also in Katz (1987), the backward-integrated chain store does not compete with the upstream monopolist in the intermediate good market as this competition will lower the input price and benefit its downstream rivals. This withdrawal behavior of an integrated firm in a vertically-related market is first found and explained in Salinger (1988).

The game in question consists of two stages. In the first stage, the input monopolist determines its optimal prices through either discriminatory or uniform pricing.[7] In the second stage, given the input price(s), the chain store and the respective local firm choose their optimal quantities in Cournot fashion in each market. We shall use backward induction to solve the subgame perfect Nash equilibrium.

Given the chain store is backward integrated, the profit functions of the local firm and the branch of the chain store in market i are as follows, respectively:

$$\pi_i^\ell = (P_i - \theta_i - w_i)q_i^\ell, \tag{1}$$

$$\pi_i^c = (P_i - v_i)q_i^c. \tag{2}$$

In the second stage, the equilibrium outputs for the local firm and the chain in market i are:

$$q_i^\ell = \frac{a - 2(\theta_i + w_i) + v_i}{3b}, \tag{3}$$

$$q_i^c = \frac{a - 2v_i + \theta_i + w_i}{3b}. \tag{4}$$

By aggregating (3) and (4), the total output in market i is derivable as follows:

$$Q_i(w_i) = \frac{2a - (v_i + \theta_i + w_i)}{3b}. \tag{5}$$

Since the chain store has its own input supply system, the derived demand for the intermediate good perceived by the upstream monopolist is the aggregation of the outputs over all the local firms, which is readily derivable from the output equilibrium in the second-stage game. With this derived demand and a given pricing regime, either being discriminatory or uniform, we can then solve for the equilibrium input price in the first-stage game in the next section.

[7] Arbitrage is assumed away in both the intermediate good and the final good markets.

3. THE INPUT MARKET EQUILIBRIUM WITH UNIFORM OR DISCRIMINATORY PRICING

In this section, we first investigate the optimal input price under uniform pricing followed by that under discriminatory pricing.

By aggregating the derived demands of each local firm, we can define the profit function of the upstream monopolist under uniform pricing Ω^u as follows:

$$\Omega^u = w^u \sum_{i=1}^{n} \frac{a - 2(\theta_i + w^u) + v_i}{3b}, \tag{6}$$

where variables with a superscript "u" denote that they are associated with the uniform pricing regime. In the uniform pricing case, the upstream monopolist chooses an input price w^u to maximize its profit Ω^u in (6). By differentiating (6) with respect to w^u, the first-order condition for profit maximization of the upstream monopolist is derivable as follows:

$$\frac{\partial \Omega^u}{\partial w^u} = \frac{na - 2\sum_{i=1}^{n} \theta_i + \sum_{i=1}^{n} v_i - 4nw^u}{3b} = 0, \tag{7}$$

and the corresponding second-order condition is satisfied as we have:

$$\frac{\partial^2 \Omega^u}{\partial w^{u2}} = -\frac{4n}{3b} < 0. \tag{8}$$

By solving (7), the equilibrium input price w^u can be derived as follows:

$$w^u = \frac{a - 2\overline{\theta} + \overline{v}}{4}. \tag{9}$$

On the other hand, if the upstream monopolist engages in price discrimination, it chooses an individual optimal input price for each local market. The profit function of the upstream monopolist is now defined as follows:

$$\Omega^d = \sum_{i=1}^{n} \frac{w_i^d[a - 2(\theta_i + w_i^d) + v_i]}{3b}, \qquad i = 1, \cdots, n, \tag{10}$$

where variables with a superscript "*d*" represent that they are associated with the discriminatory pricing regime. Maximizing the above profit function with respect to $w_1^d, w_2^d, \cdots, w_n^d$, we can derive the first-order conditions for this profit maximization as follows:

$$\frac{\partial \Omega^d}{\partial w_i^d} = \frac{a - 2\theta_i + v_i - 4w_i^d}{3b} = 0, \qquad i = 1, \cdots, n, \tag{11}$$

The second-order conditions are satisfied as we have:

$$\frac{\partial^2 \Omega^d}{\partial w_i^{d2}} = -\frac{4}{3b} < 0, \qquad i = 1, \cdots, n. \tag{12}$$

From the first-order conditions in (11), we can derive the equilibrium input price for each local firm as follows:

$$w_i^d = \frac{a - 2\theta_i + v_i}{4}, \qquad i = 1, \cdots, n. \tag{13}$$

The above equation shows that the optimal input price is negatively related to the local firm's marginal cost θ_i, while it is positively related to the marginal cost of the branch of the chain v_i. Namely, the optimal input price depends not only on the local firm's marginal cost but also its rival chain store's marginal cost. Hence, a more efficient local firm may be charged a lower price if its rival has a low marginal cost. In the literature of input price discrimination, it is generally concluded that an upstream monopolist will always charge a more efficient downstream firm a higher input price, so as to extract maximal rent from downstream firms. But this is true if input buyers do not encounter competition in their local markets. In our model, the chain store is assumed to have various marginal costs while competing with local firms in the downstream markets. A more efficient downstream firm does not necessarily pay a higher input price.

Let us compare the input prices derived under the two pricing regimes. Denote Δw_i as the difference of input prices for market i, which are derived from (9) and (13), as follows:

$$\Delta w_i = w_i^d - w^u = \frac{v_i - \overline{v} - 2(\theta_i - \overline{\theta})}{4}. \tag{14}$$

The above equation shows that the input price difference is determined by the marginal cost of local firm i, the marginal cost of the respective branch and also the average costs of the local firms and the branches of the chain. For the chain store, if $v_i - \overline{v} < 0$ then the marginal cost of branch i is lower than the average marginal cost of all the branches. This implies that the branch i is more efficient than other branches on average. Also, as $\theta_i - \overline{\theta} < 0$, the local firm i is more efficient than other local firms on average. Even though the input price is decreasing with the marginal cost of local firm i ($\partial w_i^d / \partial \theta_i < 0$ from (13)), the upstream monopolist with discriminatory pricing will charge a more efficient local firm a lower price if it's rival branch is sufficiently efficient (i.e. $(v_i - \overline{v}) < 2(\theta_i - \overline{\theta}) < 0$). According to the above discussion, we can establish the following proposition.

Proposition 1 Price discrimination by an upstream monopolist does not necessarily hurt a more efficient downstream firm.

This result is significantly different from those found in Katz (1987), DeGraba (1990) and Yoshida (2000) in which an upstream monopolist tends to charge a higher input price for a more efficient downstream firm. It is also different from the conclusion made by Inderst and Valletti (2009), who show that the more efficient input buyer always receives a price discount from a discriminatory monopolist. In their paper, other than the incumbent upstream monopolists, there exist potential input suppliers. As the more efficient downstream firm is more capable of altering its input supplier, the input monopolist is better off to charge the more efficient downstream firm a lower input price under price discrimination. Even though their paper and our paper both find that more efficient downstream firms may receive lower prices, their causes are very different. Furthermore, the intuition behind our result is also different from that in Tyagi (2001).[8] In his paper, the upstream supplier charges a large input buyer a lower price in order to lower the level of tacit collusion the downstream firms can sustain in the final good market. In our paper, whether the relatively efficient local firms pay a lower or a higher input price depends on the productivity of its rival. It implies that the cost heterogeneity within and among the final good markets plays an important role in determining the optimal prices of the upstream monopolist.

[8] We thank an anonymous referee for providing this reference.

4. OUTPUT ALLOCATION, PRODUCTION EFFICIENCY AND WELFARE

Before comparing the output allocation between the two pricing policies, we first rewrite (14) as follows:

$$\Delta w_i = \frac{v_i - 2\theta_i - (\overline{v} - 2\overline{\theta})}{4} = \frac{\eta_i - \overline{\eta}}{4}, \tag{15}$$

where $\eta_i = v_i - 2\theta_i$ and $\overline{\eta} = \overline{v} - 2\overline{\theta}$. Using (15), it is easy to show that the total output levels under the two pricing regimes are the same and we can thus establish the following lemma.

Lemma 1 With a linear demand function for the final good markets, price discrimination in the input markets does not change the total output level.

Proof: Denote ΔQ as the difference of the total output between the two pricing schemes. Using (5), we can obtain:

$$\Delta Q = \sum_{i=1}^{n} Q_i(w_i^d) - \sum_{i=1}^{n} Q_i(w^u) = \sum_{i=1}^{n} \frac{w^u - w_i^d}{3b} = \frac{-1}{3b} \sum_{i}^{n} \Delta w_i. \tag{16}$$

As the summation of Δw_i in (16) is zero by (15), the total outputs are the same under the two pricing regimes.

Similar results can be easily found in the existing price discrimination literature. But in this paper, we have shown that this result is robust even in the framework in which the chain store incurs different marginal costs while serving local markets. The intuition behind this lemma is as follows. With price discrimination, the upstream monopolist will charge input buyers different input prices. However, the average input prices under the two pricing regimes remain the same. With the same average input prices, the average marginal costs for the whole downstream industry under the two pricing regimes are equivalent. As is well known that total output under Cournot competition depends only on the average cost of the industry, with the same average marginal cost, the total outputs under the two pricing regimes are necessarily identical.

This result is important when we compare the welfare under the two pricing regimes.

To facilitate our analysis, we shall decompose the social welfare function into the following two parts: [9]

$$\text{SW} = \sum_{i=1}^{n} \text{CB}_i - \sum_{i=1}^{n} \text{PC}_i, \tag{17}$$

The first term on the RHS in (17) measures the consumption benefit from the final good whose magnitude depends on the output allocation among the local markets. The second term represents the production cost of the final good, which will be used to measure the production efficiency.

The consumption benefit in each market (CB_i), measured by the trapezoid area between the demand curve and the horizontal axis at a given output level, is derivable as follows:

$$\text{CB}_i = \frac{[a + P_i(w_i)]Q_i(w_i)}{2}. \tag{18}$$

The production cost in market i (PC_i) is equal to the sum of the production costs of the local firm (net of the input payment) and the chain store in market i which can be expressed as follows: [10]

$$\text{PC}_i = \theta_i q_i^{\ell}(w_i) + v_i q_i^{c}(w_i). \tag{19}$$

By substituting (18) and (19) into (17), we can rewrite the social welfare function as follows:

$$\text{SW} = \sum_{i=1}^{n} \frac{[a + P_i(w_i)]Q_i(w_i)}{2} - \sum_{i=1}^{n} [\theta_i q_i^{\ell}(w_i) + v_i q_i^{c}(w_i)]. \tag{20}$$

In the following analysis, we shall measure the total consumption benefits and

[9] The social welfare in our model is comprised of consumer surplus, profits of the local firms, the chain store and the upstream monopolist.

[10] For expositional convenience, we use "production cost" to represent the total production cost of a particular market and "total production cost" as the sum of the production costs of all the markets. Since the input cost is also the upstream firm's revenue, they are cancelled out when we calculate social welfare.

the total production costs under the two pricing schemes and denote their difference by ΔCB and ΔPC respectively. The social welfare ranking can be readily derived once the two differences are known. In what follows, we shall first investigate the total consumption benefit followed by the total production costs.

4.1 The Output Allocation Efficiency

In our model, consumption benefit is determined by output allocation efficiency among the local markets. By the sign of ΔCB, we can determine whether or not price discrimination enhances the output allocation efficiency. If the sign of ΔCB is positive, it implies that discriminatory pricing is superior to uniform pricing in terms of output allocation efficiency.

Using (18), we can compare the difference in output allocation efficiency between discriminatory and the uniform pricing as follows:

$$\begin{aligned}\Delta\text{CB} &= \frac{b}{2}\sum_{i=1}^{n}\left[Q_i(w^u)^2 - Q_i(w_i^d)^2\right]\\ &= \frac{b}{2}\sum_{i=1}^{n}\left[\left(Q_i(w^u)^2 - \overline{Q}^2\right) - \left(Q_i(w_i^d)^2 - \overline{Q}^2\right)\right]\\ &= \frac{nb}{2}\left\{\frac{1}{n}\sum_{i=1}^{n}\left[Q_i(w^u) - \overline{Q}\right]^2 - \frac{1}{n}\sum_{i=1}^{n}\left[Q_i(w_i^d) - \overline{Q}\right]^2\right\}\\ &= \frac{nb}{2}\left[\sigma_q^2(w^u) - \sigma_q^2(w^d)\right] > (<)\, 0, \qquad \text{if } \sigma_q^2(w^d) < (>)\, \sigma_q^2(w^u),\end{aligned} \tag{21}$$

where $\sigma_q^2(w^d)$ and $\sigma_q^2(w^u)$ denote respectively the variances of outputs in the n final good markets under the discriminatory and the uniform pricing regimes. A lower output variance implies that outputs are distributed more evenly among the downstream markets, and hence the output allocation efficiency is higher.[11] Therefore, we can conclude that the discriminatory pricing is superior to the uniform pricing in term of output allocation efficiency if and only if the outputs are more evenly distributed over the markets under the former than under the latter. The above discussion leads to the following lemma:

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[11] Because the demand curves in the final good market are identical, for any given output level, a more even output distribution means consumers' willingness to pay for the last unit of the final good is more uniform which implies higher output allocation efficiency.

Lemma 2 A lower output variance implies higher output allocation efficiency.

Lemma 2 indicates that output allocation efficiency depends on the output distribution among the final good markets. In our model, price discrimination in the input markets can improve the output allocation efficiency in the final good market. The economic intuition behind this result is as follows. Since the two types of the firms engage in Cournot competition in each final good market, the market output depends on the average marginal cost of the firms in that market. It implies that the sum of the production costs of the local firm and the chain in market i, $w_i + \theta_i + v_i$, is key in determining the output level. As we have assumed that $\theta_i + v_i \neq \theta_j + v_j$, for any $i \neq j$, the output variance of the local firms is not equal to zero under uniform pricing even if $w_i = w^u$. This implies that input price discrimination improves the output allocation efficiency if the output variance becomes lower. We can use a two-market case to demonstrate this result. Given $\theta_1 + v_1 < \theta_2 + v_2$, we have $\Delta Q^u = Q_1^u - Q_2^u > 0$ in the case of uniform pricing. If $2(\theta_1 - \theta_2) < v_1 - v_2 < 2(\theta_2 - \theta_1)/5$, it is found that $\theta_1 + v_1 + w_1^d < \theta_2 + v_2 + w_2^d$ and $\Delta Q^d = Q_1^d - Q_2^d > 0$ under price discrimination. However, it is straightforward to show that $\Delta Q^d < \Delta Q^u$ as $w_1^d > w^u > w_2^d$. This result implies that the output difference and the output variance are both smaller under discriminatory pricing than uniform pricing. From the above discussions, we can establish the following proposition:

Proposition 2 Input price discrimination could result in a more even output distribution among the final good markets, and higher consumption benefit which is in favor of social welfare.

4.2 The Production Efficiency

We have examined the output distribution among the final good markets which determines the output allocation efficiency and the resulting consumer benefit in the welfare function. Now we turn to examine the effect of price discrimination on the total production cost which is the second part of the welfare function.

By (19), we can derive the difference in total production costs between the discriminatory and uniform pricing as follows:

$$\Delta \text{PC} = \sum_{i=1}^{n} \theta_i [q_i^{\ell}(w_i^d) - q_i^{\ell}(w^u)] + \sum_{i=1}^{n} v_i [q_i^c(w_i^d) - q_i^c(w^u)], \tag{22}$$

where the first-term on the RHS of (22) is the difference in production costs of the local firms between the two pricing schemes and the second-term is that of the chain store. By using (3), (4), (15), (22), the two cost differences can be derived as follows:

$$\sum_{i=1}^{n} \theta_i[q_i^{\ell}(w_i^d) - q_i^{\ell}(w^u)] = -\frac{1}{12b} \sum_{i=1}^{n} 2\theta_i(\eta_i - \overline{\eta}), \tag{23}$$

$$\sum_{i=1}^{n} v_i[q_i^{c}(w_i^d) - q_i^{c}(w^u)] = \frac{1}{12b} \sum_{i=1}^{n} v_i(\eta_i - \overline{\eta}). \tag{24}$$

It is obvious that (23) and (24) are opposite in sign. By substituting (23) and (24) into (22), we have:

$$\Delta \text{PC} = \frac{1}{12b} \sum_{i=1}^{n} (\eta_i - \overline{\eta})\eta_i = \frac{1}{12b} \sum_{i=1}^{n} (\eta_i - \overline{\eta})^2 > 0. \tag{25}$$

It is found that the total production cost is higher under price discrimination. Based on this finding, we can establish the following proposition:

Proposition 3 Price discrimination in input markets necessarily lowers production efficiency of the final good which is detrimental to social welfare.

This result is in line with the finding in Yoshida (2000). In a model with one upstream monopolist selling its input to two downstream firms, he shows that the input monopolist tends to charge the more efficient downstream firm a higher price under discriminatory pricing than under uniform pricing, causing a loss in production efficiency. This result holds true even if each local firm competes with a chain store. But the cause of the result is different. In our model, the total production cost is comprised of the production costs of local firms and that of the chain store. With price discrimination, the upstream monopolist charges local firms different prices, which affects not only the output of the local firms, but also the output of the backward-integrated chain. Under discriminatory pricing, there are two opposite effects as shown in (23) and (24), jointly affecting the production efficiency. If the upstream monopolist charges a more efficient local firm a lower input price, it entails a positive effect arising from the *decreased* production cost of the local firms. It also entails a negative effect as it decreases the output of the more efficient branch and thereby *increases* the production cost of the chain store. From (25), we find that the overall production cost necessarily increases. That is to say, the production efficiency is necessarily lower under input price discrimination.

4.3 Welfare Effect of Price Discrimination

As we mentioned before, input price discrimination generates two welfare effects: the output allocation efficiency effect and the production efficiency effect. The first effect shows that price discrimination is welfare-improving if it leads to a more even output distribution. The latter on the other hand indicates that price discrimination is welfare-deteriorating as it lowers the total production efficiency. In order to compare these two effects, we need to work out the reduced form of the welfare change.

By substituting (9) and (13) into (21) and rearranging the terms, we obtain:

$$\Delta\text{CB} = \frac{1}{288b}\sum_{i=1}^{n}(\eta_i - \overline{\eta})(-9v_i - 6\theta_i). \tag{26}$$

Then, by (25) and (26), the change in social welfare after price discrimination is given as follows:

$$\begin{aligned}\Delta\text{SW} &= \Delta\text{CB} - \Delta\text{PC} \\ &= \frac{1}{96b}\sum_{i=1}^{n}(\eta_i - \overline{\eta})(14\theta_i - 11v_i). \end{aligned}\tag{27}$$

To gain more insight, we measure the welfare change in terms of the variances and the covariance of v_i and θ_i as follows:

$$\Delta\text{CB} = \frac{n}{96b}(-3\sigma_v^2 + 4\sigma_{v\theta} + 4\sigma_\theta^2), \tag{28}$$

$$\Delta\text{PC} = \frac{n}{96b}(8\sigma_v^2 - 32\sigma_{v\theta} + 32\sigma_\theta^2), \tag{29}$$

$$\Delta\text{SW} = \frac{n}{96b}(-11\sigma_v^2 + 36\sigma_{v\theta} - 28\sigma_\theta^2). \tag{30}$$

Equation (30) indicates that the welfare change depends on the distributions of v_i and θ_i and their covariance. If either cost variance increases, social welfare under price discrimination declines because it results in a higher production cost. Furthermore, if the covariance is negative in sign, social welfare is necessarily lower under price discrimination. However, if there is a positive covariance of v_i and θ_i, price discrimination could enhance social welfare as it results in a higher consumption benefit and a lower production cost by (28) and (29).

To have a clear solution for the welfare change, let us reduce the number of the local markets to 2. From this simplified model, we can prove that input price discrimination is welfare-improving under certain circumstances.

Let n be equal to 2, the cost variances and covariance are derivable as follows:

$$\sigma_v^2 = \frac{1}{2}\sum_{i=1}^{2}(v_i - \overline{v})^2 = \frac{1}{4}(v_2 - v_1)^2,$$

$$\sigma_\theta^2 = \frac{1}{2}\sum_{i=1}^{2}(\theta_i - \overline{\theta})^2 = \frac{1}{4}(\theta_2 - \theta_1)^2,$$

$$\sigma_{v\theta} = \frac{1}{2}\sum_{i=1}^{2}(v_i - \overline{v})(\theta_i - \overline{\theta}) = \frac{1}{4}(v_2 - v_1)(\theta_2 - \theta_1).$$

Using the above equations, we can rewrite (28), (29) and (30) as follows:

$$\Delta\text{CB} = \frac{[2(\theta_2 - \theta_1) + 3(v_2 - v_1)][2(\theta_2 - \theta_1) - (v_2 - v_1)]}{192b}, \tag{31}$$

$$\Delta\text{PC} = \frac{[2(\theta_2 - \theta_1) - (v_2 - v_1)]^2}{24b} > 0, \tag{32}$$

and

$$\Delta\text{SW} = \frac{[11(v_2 - v_1) - 14(\theta_2 - \theta_1)][2(\theta_2 - \theta_1) - (v_2 - v_1)]}{192b}. \tag{33}$$

Denote $\Delta\theta$ and Δv as $\theta_2 - \theta_1$ and $v_2 - v_1$, respectively. Without loss of generality, we further assume that local firm 1 is more efficient than local firm 2 so that we have $\Delta\theta > 0$. According to (33), the sign of ΔSW now depends on the values of $\Delta\theta$ and Δv. Specifically, with two local markets, input price discrimination can improve social welfare if and only if $\Delta v/2 < \Delta\theta < 11\Delta v/14$. Thus, we summarize the above results into the following proposition:

Proposition 4 When the chain store is backward-integrated, price discrimination in input markets enhances social welfare if it leads to a more even output distribution in the final good markets, and the increased consumer benefit outweighs the increased production cost.

Katz (1987), DeGraba (1990) and Yoshida (2000) show that input price discrimination lowers production efficiency and thus reduces welfare. In contrast, we have found a counter-example in which input price discrimination can raise social welfare even though it reduces production efficiency of the final good. It is worth noting that our welfare outcome is same as that in Inderst and Valletti (2009). They incorporate potential input suppliers into their model and find that price discrimination may also enhance social welfare. Even though the outcomes of the welfare in the two papers are the same, the causes are very different. There is no potential input supplier in our paper and the higher social welfare is attributed to the improved output allocation which is not concerned in their paper.

5. EXTENSIONS

In this section, we extend our model by allowing the chain store to operate in the intermediate good markets. That is, the chain store can be either buy or sell the intermediate good. We shall consider two cases in the following two subsections. In Section 5.1, we shall assume that the chain store is not backward integrated and has to buy the intermediate good from the upstream monopolist. In Section 5.2, we assume the chain store competes with the incumbent intermediate good supplier in the intermediate good markets.[12]

5.1 Price Discrimination without Backward Integration

In previous sections, we have assumed the chain store is backward integrated and found that price discrimination in input markets, even with diminished production efficiency, may still improve social welfare. In this subsection, we shall examine if this result is still valid if the chain store is not backward integrated.

All the assumptions and notations made in previous sections remain effective in this subsection. Due to the setting of this subsection, we introduce a new variable w_c which denotes the price of the intermediate good paid by the chain store who is now has to buy the input from the upstream monopolist. With the same game structure

[12] We are indebted to a referee for suggesting this modification.

as before, the upstream monopolist determines the optimal input prices through either discriminatory pricing or uniform pricing in the first-stage game. Given the input prices, the chain store and the local firm determine their outputs in Cournot fashion in the second-stage game. As usual, we follow backward induction to solve the second stage of the game first.

From the second-stage equilibrium, we can obtain the derived demand curves for the intermediate good of local firms as follows:

$$q_i^\ell = \frac{a + (w_c + v_i) - 2(w_i + \theta_i)}{3b}, \qquad i = 1, \cdots, n.$$

The derived demand curve of the intermediate good for the chain store is:

$$q^c = \sum_{i=1}^{n} q_i^c = \frac{n(a - 2w_c) + \sum_{i=1}^{n}(w_i + \theta_i) - 2\sum_{i=1}^{n} v_i}{3b}.$$

According to the derived demand curves, the upstream monopolist chooses the input prices to maximize the following profit function:

$$\Omega = w_c q^c + \sum_{i=1}^{n} w_i q_i^\ell.$$

In the case of uniform pricing where each downstream firm pays the same input price (w^u), the equilibrium input price is derivable as follows:

$$w^u = \frac{a - (\overline{\theta} + \overline{v})}{4}.$$

In the case of price discrimination, the equilibrium prices for each downstream firm are derivable as follows:

$$w_i^d = \frac{a - \theta_i}{2} + \frac{v_i - \overline{v}}{4}, \qquad i \in \{1, 2, \cdots, n\},$$

and

Table 1 Numerical Examples of the Welfare Difference

$(v_1, v_2, \theta_1, \theta_2)$	ΔCB	ΔPC	ΔSW
(0,8,2,7)	0.354	0.292	0.063
(0,9,2,7)	0.193	0.042	0.151
(0,10,2,7)	0.000	0.125	−0.125
(0,11,2,7)	−0.224	0.542	−0.766

Note: The value of b is assumed to be unity in the numerical examples.

$$w_c^d = \frac{a-\overline{v}}{2}.$$

For easy analysis and illustration, let us assume that there are only two independent local markets, which are denoted as market 1 and market 2. Proceeding as in Section 4, we can derive the differences of the total consumption benefit and the total production cost under the two pricing regimes as follows:

$$\Delta \mathrm{CB} = \frac{[(\theta_2-\theta_1)-(v_2-v_1)][3(v_2-v_1)+2(\theta_2-\theta_1)]}{192b},$$
$$\Delta \mathrm{PC} = \frac{4(v_1^2+v_2^2)+7(\theta_1^2+\theta_2^2)-10(v_1\theta_1+v_2\theta_2)-2(v_1\theta_2+v_2\theta_1+\theta_1\theta_2)}{24b}.$$

From the above two equations, we can further derive the difference in welfare between the two pricing policies as follows:

$$\begin{aligned}\Delta \mathrm{SW} &= \Delta \mathrm{CB} - \Delta \mathrm{PC}\\ &= \frac{1}{192b}[v_1(84\theta_1+12\theta_2-35v_1+3v_2)\\ &\qquad + v_2(12\theta_1+84\theta_2-35v_2+3v_1)-4(13\theta_1^2+13\theta_2^2-2\theta_1\theta_2)].\end{aligned}$$

As before, the sign of the welfare difference (ΔSW) depends on the marginal costs of the local firms and the branches. Some numerical examples are reported in Table 1.

From the above table, one can find that price discrimination in input markets always lowers production efficiency due to the increase in total production cost. However, it can enhance social welfare as the increase in total consumption benefit outweighs the increase in production cost. It implies that social welfare may still be higher under price discrimination even though the chain store is not backward integrated. Namely, our results from Propositions 2 to 4 are robust regardless of the chain store being backward integrated or not.

5.2 Competition in the Intermediate Good Market

In this subsection, we examine the case where the chain store competes with the upstream incumbent in the intermediate good markets.[13] To compare the outcomes under the two pricing policies and also to compare finding of this subsection with those derived in previous sections, we shall follow the setting in previous sections to assume the chain store and the incumbent upstream firm compete in terms of prices in the intermediate good market.[14]

As before, we shall examine two cases. In the case of uniform pricing, each firm charges an identical input price to all local firms. Namely, the two input producers compete in a Bertrand fashion, given the aggregate derived demands from the local firms. The profit functions of the chain store and the incumbent upstream firm are as follows:

$$\pi_i^c = (P_i - v_i)q_i^c + w^{uc}x^c,$$
$$\Omega = w^u Q,$$

where w^{uc} is the input price charged by the chain store; Ω and x^c are respectively the sales of the incumbent input supplier and the chain store to the intermediate markets.

In the case of discriminatory pricing, the incumbent input supplier and the chain compete in Bertrand fashion in each downstream market. The profit functions of the chain store and the incumbent upstream firm are now specified as follows:

$$\pi_i^c = (P_i - v_i)q_i^c + \sum_{i=1}^{n} w_i^{dc} x_i^c,$$

[13] We are indebted to a referee for suggesting this extension.

[14] In the literature with regard to successive oligopoly, several papers have assumed vertically-integrated firms and upstream firms compete in a Cournot fashion. See for example, Higgins (1999), Schrader and Martin (1998) and Wang et al. (2005). We do not follow this fashion as the equilibrium for input price discrimination is hard to derive under the current setting.

$$\Omega = \sum_{i=1}^{n} w_i^d q_i,$$

where w^{dc} is the input price charged by the chain store, q_i and x_i^c are respectively the output of the upstream firm and the chain store sold to the local firm in market i. Since the intermediate good is homogeneous, it is intuitive to conclude that the upstream firm and the chain will undercut each other's price until the input price is equal to the marginal cost of the less efficient input supplier. Given that the marginal costs of the chain and the incumbent are assumed to be nil, the equilibrium input prices are zero under the two pricing schemes. It is then straightforward to show that the total output, consumer surplus, total profits for firms and the resulting welfare levels are all the same under the two pricing schemes.

6. CONCLUDING REMARKS

The relative merits of uniform pricing and discriminatory pricing have been studied extensively ever since the Robinson-Patman Act was enforced by the U.S. government. Robinson (1933) first proposed that the total output is not changed under price discrimination if the final good demands are linear. Most of the studies along this line have concluded that price discrimination in final goods or intermediate goods is adverse to social welfare.

By employing a chain-store model *à la* Katz (1987) but assuming different marginal costs among the retailers and the branches of the chain store, this paper has investigated the welfare implications of third-degree price discrimination by an upstream monopolist who produces and sells an intermediate good to downstream markets. Our assumption on the marginal costs not only generalizes the model pioneered by Katz (1987) but also highlights an interesting case in which *input price discrimination may improve output allocation efficiency in the final good market and increases social welfare*.

The major findings of this paper are summarized as follows. In our chain store model, the upstream monopolist may charge a more efficient retailer a lower input price. This phenomenon is in contradiction to that prevailing in the literature. Moreover, production efficiency of the final good necessarily decreases under price discrimination, but social welfare can still increase. This result runs counter to the literature concerning input price discrimination. For example, Katz (1987), DeGraba (1990) and Yoshida (2000) have argued that input price discrimination is welfare-deteriorated as it tends to reduce production efficiency through shifting the productions of the final good from efficient producers to inefficient producers. This is, of course, because they have all ignored the output allocation efficiency effect addressed by this paper. In addition, the above findings are robust regardless of the chain store being backward integrated or not. Finally, if we allow the backward integrated chain store to operate in the intermediate good market by competing with the incumbent upstream firm in a Bertrand fashion, social welfare is indifferent between the two pricing schemes.

Moreover, Schmalensee (1981) and Varian (1985) both show that price discrimination in a final good market is welfare-improving only if it leads to a higher output. In this paper, although the total output remains unchanged, social welfare can still be improved under price discrimination. It also implies that their result which was derived from a final good model cannot be applied to models dealing with input markets.

Our finding bears a policy implication. Since price discrimination may improve social welfare, it implies that a blanket prohibition of third degree price discrimination is not socially desirable.

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社會福利, 產出配置與要素市場差別訂價

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關鍵詞: 要素差別訂價、社會福利、產出配置
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* 聯繫作者: 彭正浩, 輔仁大學經濟學系, 新北市 242 新莊區中正路 510 號。電話: (02) 2905-2876; 傳真: (02) 2905-2188; E-mail: chpon@mail.fju.edu.tw。本文曾發表於國立台灣大學社會科學院國際貿易研討會與台灣經濟學會 2010 年年會。與會學者對本文之批評與建議, 使我們獲益良多, 在此謹致謝忱。同時感謝本刊編輯委員及兩位匿名審查委員惠賜改進意見, 使本文更臻完善。文中如有任何缺失, 悉由作者負責。

摘　要

本研究分析上游獨占廠商訂價決策對最終財產量分配及社會福利的影響。假設要素市場存在一獨占廠商將其生產的中間財銷售給下游的多家獨立零售商(local retailer)生產最終財。每一個最終財市場中存在一家獨立零售廠商與一家向後整合的連鎖廠商(backward-integrated chain store)進行數量競爭。不同於過去文獻的結論,本文發現上游獨占廠商在採取差別訂價時對高效率的零售廠商可能收取較低的要素價格。此外,我們也發現當配置效率的正效果高於生產效率的負效果時,即使總產量不變,要素差別訂價亦有可能提高社會福利。本文的發現與一般文獻上認為中間財差別訂價有損社會福利的看法有很大的不同。且此一結論在連鎖廠商沒有向後整合的情形之下仍然成立。