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Market Concentration and Licensing Royalty in an Asymmetric Oligopoly

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ABSTRACT

This paper studies the interaction of market concentration and licensing royalty for an innovator following RAND terms under an asymmetric oligopoly in which the technology transfer takes place from the innovator to firms having asymmetric cost. Taking cost variance as a proxy to evaluate market concentration, this paper shows that the nexus of market concentration and licensing royalty is highly sensitive to the curvature of market demand. When market demand is concave (convex) to the origin, the nexus of market concentration and royalty rate is positive (negative), whereas in the case of linear demand the royalty is independent of market concentration. As a result, market concentration and market demand crucially affect the licensing royalty.

1. INTRODUCTION

Patent licensing provides an opportunity for innovators to realize profits associated with their effort related to research and development.[1] Ever since the seminal paper of Kamien and Tauman (1984), scholars have offered numerous insights to increase our knowledge on licensing royalty. Even though many scholars assert that two-part licensing is no worse than just a fixed fee or a royalty, it is still debatable whether the optimal licensing strategy is a fixed fee, a royalty, or a two-part licensing fee (Kamien and Tauman, 1984, 1986; Katz and Shapiro, 1986; Kamien, 1992; Poddar and Sinha, 2010; Chang et al., 2013; Wang et al., 2013; Battersby and Grimes, 2015, among others).[2]

This paper investigates how the degree of market concentration interacts with licensing royalty following reasonable and non-discriminatory (RAND) terms in an asymmetric oligopoly, where one outsider patentee licenses its technology to n firms.[3] Tracing the theoretical literature on licensing, we find few suggestions indicating how a licensor behaves in response to different degrees of market concentration. Should a licensor in a more concentrated market always charge a higher royalty, because a concentrated market is more profitable and therefore the licensor can charge a higher royalty and thus extract more rent from the market? Indeed, the empirical literature on market concentration has explicitly pointed out how market concentration could influence industrial behaviors (Stavins, 2001; Peria and Mody, 2004; Silva et al., 2015; Yang and Tsou, 2016, among others). Acs and Audretsch (1988) point out a negative relationship between market concentration and innovation behavior. Park et al. (2011)

[1] Patent licensing is pervasive, taking place in most industries. Total transactions of technological sales and import payments for technology purchases from 1970 to 1988 increased fourfold in Japan and the U.K. (Nadiri, 1993). In 2002, U.S. companies collected $29.023 billion in royalties and fees from foreign subsidiaries and $12.075 billion more from unaffiliated firms in foreign lands (Vishwasrao, 2007).

[2] Poddar and Sinha (2010) contend that the two-part licensing strategy is optimal when the cost difference between the firms is moderate in a standardized oligopolistic model. Rockett (1990) concludes that a contract with royalties is preferred under non-drastic cost-reducing process innovations. These results are in stark contrast to Kamien and Tauman (1984), who show that a fixed fee is preferred to royalties. As seen in the literature, the optimal licensing strategy and relevant issues are still debatable (see Villalonga, 2000; Norback and Persson, 2004; Lee and Quayes, 2005).

[3] RAND denotes reasonable and non-discriminatory terms. Non-discriminatory relates to the terms and rates that are included in licensing agreements. This commitment requires that licensors treat each individual licensee in a similar manner. This does not mean that the payment terms cannot change depending on the volume and creditworthiness of the licensee. See http://itlaw.wikia.com/wiki/F/RAND.

study the effects of licensing fees and auctions for the 3G spectrum on consumer prices and the market structure using data from the mobile markets of 21 OECD countries.[4] Yang and Tsou (2016) find that there is an increasing but concave relationship between market concentration and R&D propensity for small- and medium-sized enterprises, but not for large firms.

The thoughts above motivate us to examine whether a higher degree of market concentration measured by the Herfindahl-Hirschman Index should involve a higher royalty. In other words, because the existing literature sheds relatively limited light on the possible interaction between licensing royalty and market concentration, we want to look at how a pervasively practiced licensing contract, namely the two-part tariff contract, is related to market concentration in an asymmetric oligopoly.[5]

Taking cost variance as a proxy to evaluate market concentration, the paper shows that the link between the market concentration index and licensing royalty rate is highly sensitive to the factor of market demand – specifically, the curvature of the market demand. We have not encountered any theoretical work that sheds light on the relationship between market concentration and licensing royalty.[6] Thus, this paper provides integral analysis that fills the gap in the existing literature by pointing out the missing link between the two.

One possible reason as to why this link has rarely been examined by scholars could be that many studies, if not most, in the licensing literature entail the domain of linear market demand (see Kamien and Tauman, 1986; Eswaran, 1994a, 1994b; Fauli-Oller and Sandonis, 2002; Fosfuri and Roca, 2004; Poddar and Sinha, 2010; Ishida et al., 2011).[7] The setting of a linear market demand leaves a relatively narrow gateway

[4] Park et al. (2011) examine certain negative perceptions, such as auctions that may cause high licensing fees, high consumer prices, and concerns about market concentration. The estimation results show no evidence to support claims of negative effects of spectrum auctions in the mobile communications market.

[5] Rostocker (1984) finds that a royalty alone is used 39% of the time, a fixed fee alone 13%, and both instruments 46%, implying that a royalty rate makes up 85% of licensing packages.

[6] According to Poddar and Sinha (2008), having "derived some testable hypotheses... [we] expect to observe only fixed fee or only royalty contract in the boom period or during big fluctuation of market demand; whereas a combination of fixed fee and royalty or only fixed fee should be observed during bust or when there is less variation in market demand." This serves as a remark on the possible relationship between a licensing strategy and variations in the consumer market.

[7] After Kamien and Tauman (1986)'s pioneer work, Eswaran (1994a, 1994b) hypothesizes that firms promote collusion by cross-licensing their competing patents and examines how an incumbent threatened by a new entry can exploit its first-mover advantage by licensing its technology to outsider firms. Fauli-Oller and Sandonis (2002) characterize situations in which licensing is a cost-reducing innovation to a rival using two-part contracts. Fosfuri and Roca (2004) examine the presumption behind the traditional wisdom that licensing through a royalty may be superior to licensing by means of a fixed fee. Mukherjee

for the various types of market demand that affect licensing behaviors. Although it is necessary to conduct relevant studies on linear demand, thus laying a solid foundation for understanding a firm's licensing behavior, it is not beyond one's imagination to realize how versatile the demand pattern of consumers turns out to be in the real business world.[8] In addition, to construct the index of market concentration, it is necessary to depict a heterogeneous environment; thus, we employ conventional wisdom to depict this asymmetric characteristic by asymmetric costs (see Collie, 1993; Neary, 1994; Long and Soubeyran, 1997; Melitz, 2003, among others).[9] Incorporating asymmetric characteristics also offers a more vivid life for the studied firms.

It is shown that the relationship between the market concentration index and the licensing royalty rate is highly sensitive to the curvature of market demand. Under linear demand, the royalty is not sensitive to market concentration at all, whereas a higher market concentration leads to a larger (smaller) royalty when market demand is concave (convex) to the origin.

The remainder of this paper is organized as follows. Section 2 displays an oligopolistic model in which one patentee licenses its technology to a batch of firms having different costs. Section 3 investigates the relationship between market concentration and licensing royalty. Section 4 demonstrates the cases of discriminatory two-part tariff and exclusive licensing. Section 5 offers concluding remarks.

2. THE MODEL

Consider that an outsider patentee with full bargaining power licenses its know-how of a patented good to n firms (hereby, the licensees) that compete with one another

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and Mukherjee (2005) show the effects of entry by a foreign firm on domestic welfare in the presence of licensing. Lin and Kulatilaka (2006) find that the intensity of the network effect can shift the choice of licensing mechanism from a royalty-based to a fee-based regime.

[8] While the setting of non-linear demand is not rare in the literature (see Bergstrom and Varian, 1985; Brander and Spencer, 1985; Aguirre et al., 2010), works that analyze the licensing issue under non-linear marked demand are relatively limited.

[9] Collie (1993) analyzes a country's strategic trade policy in an asymmetric setting and concludes that there is a rationalization effect, which does not occur under a symmetric oligopoly. Neary (1994) argues that subsidies are optimal only for very low values of social cost of public funds in the setting in which home and foreign firms have different costs. Long and Soubeyran (1997) show that the interplay between the concentration index and the elasticity of the demand curve has major importance in determining optimal trade policies, if domestic firms do not have identical unit costs. Melitz (2003) develops a dynamic industry model and analyzes intra-industry trade for heterogeneous firms.

in the global market *a la* Cournot fashion.[10] The global firms request the specific technological know-how in order to produce the patented good in the market; [11] that is, without the licenses, no other firms could produce the good even though the firm has the capability of producing the product. However, the patentee should license to any firm that is willing to enter this market without a discriminatory royalty rate in order to follow the commonly practiced rules that govern the ownership of patent rights, such as reasonable and non-discriminatory terms (abbreviated as RAND) or fair, reasonable and non-discriminatory terms (abbreviated as FRAND).[12]

To begin with, the n firms are endowed with different production technologies. The asymmetric characteristics of the different production technologies are idiosyncratic and are reflected by different marginal costs. We exogenously present the cost of firm i before licensing occurs as $\underline{c}_i$. Once licensee i receives the new technology, the firm's marginal production cost decreases by ε, and the marginal cost becomes $\underline{c}_i - \varepsilon$. Let the inverse market demand be $P = P(Q)$, where $Q = \sum_1^n q_i$ and q_i stands for the output of firm i.

We employ a two-stage game to describe the scenario. In the first stage, the patentee determines a package of licensing strategies that consists of a non-discriminatory royalty rate per unit of output and a fixed fee collected from n firms in order to maximize its own profit as follows:

$$\pi = r \sum_1^n q_i + \sum_1^n F_i. \tag{1}$$

[10] We note the concept of the product life-cycle theory, whereby new products appear, then mature, and eventually die off (Vernon, 1979). The global market usually adopts and uses a product, typically through mass production and after the skill-advanced home patentee has licensed the technological know-how to enterprises worldwide. Moreover, there are various ways to determine the royalty rate. For instance, the royalty rate can be derived by a Nash bargaining game between the patentee and the licensees. Even though the royalty rate in this paper is determined by the patentee in pursuit of profit maximization, setting the bargaining power of the patentee in the Nash bargaining game to one corresponds to the setting of this paper.

[11] For example, pervasive licensing contributes to the growth of technology industry. "Invention ... the fact that technology licensing remains a $150 billion annual business today. In fact, the smartphone industry's unprecedentedly-rapid growth would have been impossible without the pervasive licensing and cross-licensing of technology (by manufacturers and non-manufacturers alike) across four different industries: mobile phones, electronics, computing, and software." See http://www.ipwatchdog.com/2015/03/29/patent-licensing-is-as-american-as-apple-pie/id=56217/.

[12] A famous case regarding FRAND occurred in 2013 when Microsoft sued Motorola for failing to license its standard-essential wi-fi and video-coding patents to Microsoft for a FRAND rate. Motorola had asked for 2.25% of the price of every Microsoft Xbox, smartphone, or other product that implemented its SEP patents. Microsoft alleged that Motorola had breached its FRAND commitments. See http://www.worldipreview.com/article/seps-in-europe-and-the-us-a-primer.

In (1), r denotes the royalty charged to licensees for each unit of production, and F_i is the fixed fee that licensee i pays to the patentee. The potential licensees decide whether to accept or reject the patentee's offer. The firm that accepts the offer becomes a licensee, while the firm that rejects hardly survives in the market and obtains zero profit. This outside option implies no know-how and no production.

It is common practice to collect the royalty on the basis of shipments after licensees pay an upfront, fixed fee negotiated with the patentee. The royalty rate is identical for all firms without discrimination according to FRAND, while the firm-specific upfront fee in the contract is not disclosed to the public most of the time.[13] The firm-specific fixed fee could depend on the effort involved with technology transfer, such that a firm with more R&D investment might pay less than one investing less in R&D. For instance, Qualcomm, a patent giant on mobile communication technology, makes chips and mainly targets end-product manufacturers. Qualcomm charges the end-produce manufacturers an upfront fee negotiated by relevant parties for the right to use the chips, followed by a certain percentage of the royalty fee according to the shipping volume in the mass production of wafers or products, implying that the end-product manufacturers need to pay a percentage fee once again to Qualcomm for every unit of product sold. In 2010, business revenue for Qualcomm Technology Licensing, which is the unit handling patent licensing for Qualcomm, reached USD \$3.66 billion, about one third of Qualcomm's total revenue.[14] The end-product manufacturers include all smartphone brands, such as Nokia, Motorola, HTC, Samsung, LG, Huawei, ZTE, etc.. Qualcomm takes USD \$1 billion as an upfront fee and also charges a royalty of USD \$40 for every iPhone Apple sold. In 3G and 4G, Qualcomm has been in the dominant position of CDMA standards development. Qualcomm charges manufacturers a rate of 2.275% royalty for 5G mobile phones and 3.25% for multi-mode 3G/4G/5G after a firm negotiates the upfront payment and signs the licensing contract with Qualcomm.[15]

The above case shows that, in order not to infringe intellectual property rights, a firm has to be licensed the associated know-how when producing a patented product,

[13] The upfront fee or the exact rate of royalty is sometimes observable when lawsuit is involved. See the case of Philips' CD-R and CD-W, http://www.zoomlaw.net/files/16-1138-12760.php and https://www.ithome.com.tw/node/12240, and the case of Qualcomm and Apple, http://technews.tw/2017/02/13/iphone-money-qualcomm/.

[14] The case is translated from a 2010 ITRI report, see http://std-share.itri.org.tw.

[15] See http://technews.tw/2017/02/13/iphone-money-qualcomm/ and https://www.pixpo.net/post416362.

even though the firm has technological capabilities to generate the know-how on its own. We list this as Assumption 1 below.

Assumption 1 (no know-how, no production) Licensee-to-be firms that reject the licensing offer obtain zero profit.

Moreover, compulsory licensing, requesting that the owner of a patent or copyright license the use of their rights in a time of national emergency or for public non-commercial use, happens occasionally, especially in the pharmaceutical industry. For instance, Taiwan issued a compulsory license in 2005 for the generic production of Tamiflu, the only drug currently available for the treatment of avian flu, in order to ensure that Taiwan has sufficient quantities of the medicine in the event of a pandemic. Indeed, patent licensing could be achieved through government involvement; however, it is common among industry participants that produce standards to license a patent, particularly standard essential patents, through private initiatives or collaboration. For instance, while Apple does not license its operating system to any cell phone manufacturers, Google's Android operation system is open to all cell phone manufacturers.[16]

Another example comes from the Voluntary Standard-setting organizations, which requires that industry participants in a standard essential patent declare that licenses are available on FRAND terms. For example, the Institute of Electrical and Electronics Engineers (IEEE) has added provisions in their amended IEEE Policy, requiring the standard essential patent owner to offer a license under FRAND terms to an unrestricted number of licensees. Here we assert that the patent is licensed to all firms, such as the listed case of open license or compulsory licensing.[17] We refer to this as Assumption 2 to facilitate brevity in writing.

Assumption 2 The patentee licenses to all firms.

In the second stage of the game, n firms engage in Cournot competition. The objective function for the licensees after the two-part licensing tariff is as follows:

$$\Psi_i = Pq_i - c_i q_i - rq_i - F_i, \quad i = 1, 2, \cdots, n. \tag{2}$$

[16] Microsoft also runs the Open License Program that allows corporate, academic, charitable, or government organizations to obtain volume licenses for Microsoft products. See https://www.microsoft.com/en-us/licensing/licensing-programs/open-license.aspx.

[17] Source: http://www.wiggin.com/15757.

In (2), $c_i \equiv \underline{c}_i - \varepsilon$ stands for the marginal production cost after technology transfer.

The subgame perfect Nash equilibrium is derived by solving backwards. The second-stage equilibrium is derivable by the profit maximization of the n firms as follows:

$$\frac{\partial \Psi_i}{\partial q_i} = (P - c_i - r) + P' q_i = 0, \quad i = 1, 2, \cdots, n. \tag{3}$$

In (3), we obtain the optimal output for licensee i as $q_i^* = (P - c_i - r)/(-P')$. Substituting the optimal output back into (2) obtains $\Psi_i = -P' q_i^{*2} - F_i$. A firm must make positive profit, which is larger than its outside option in Assumption 1, after taking the licensing offer in order to survive; otherwise, it rejects the offer and leaves the market. As a result, firm i, which produces $q_i \geq \underline{q}_i$, where $\underline{q}_i \equiv \sqrt{-F_i/P'}$, enters the market.

Adding up (3) yields:

$$nP - \sum_1^n c_i - nr + P'Q = 0. \tag{4}$$

Equation (4) shows that the total output is determined by specific factors, including the sum of the marginal production cost for firms – i.e., $\sum_1^n c_i$, the royalty, and the number of firms. Given the knowledge provided by Bergstrom and Varian (1985) and further applied by Long and Soubeyran (1997), the Cournot equilibrium output of the industry is a function of the average marginal production cost and is independent of the distribution of marginal production cost among firms.

By taking licensing strategy into consideration, the result of Bergstrom and Varian (1985) is still sustained. The distribution of the marginal cost for firms, ceteris paribus, has no impact on total industry output as long as the sum of the marginal cost for these firms stays constant. The market concentration index, which is measured by the Herfindahl-Hirschman Index (H), is defined by the square sum of each firm's market share – i.e. $H = \sum_1^n q_i^2/Q^2$ – and is further developed as follows: [18]

$$H = \frac{(P - \bar{c}_n - r)^2 + \text{Var}(c_i)}{n(P - \bar{c}_n - r)^2}, \tag{5}$$

[18] The index is between zero and one. The higher the Herfindahl-Hirschman Index is, the more concentrated the market is. See Appendix 1 for the derivation of H.

where $\overline{c}_n \equiv \sum_1^n c_i/n$, and $\text{Var}(c_i) \equiv \sum_1^n (c_i - \overline{c}_n)^2/n$ represents the cost variance of the firms. Here, $(P - \overline{c}_n - r)$ is the average mark-up for the firms. The market concentration index in (5) depends on factors such as the number, the cost variance, and the average mark-up of the firms.

The cost of firms can capture the heterogeneity feature or the asymmetric structure of the firms, and therefore relevant studies often employ asymmetry in firm costs to examine the effects associated with firm asymmetry (see footnote 9). From the knowledge set forth by Bergstrom and Varian (1985), we observe that, ceteris paribus, a higher cost variance leads to a higher market concentration without altering total output – i.e., $\partial H/\partial \text{Var}(c_i) = 1/[n(P - \overline{c}_n - r)^2] > 0$.

The cost variance serves as a modest proxy for the market concentration index due to the following two reasons. First, $\text{Var}(c_i)$ is exogenous in the model that is not associated with the royalty rate. Second, the cost variance is a crucial factor in the market concentration ratio in the sense that firms' higher cost variation, without altering total output, leads to a higher degree of market concentration. We note in particular that, despite $\text{Var}(c_i)$ being used as the proxy to evaluate market concentration, there are differences between H and $\text{Var}(c_i)$, e.g., the cost variance is one of the factors that alter the degree of market concentration.

By totally differentiating (4), we obtain the comparative statics of the royalty on output as follows: [19]

$$\frac{dQ}{dr} = \frac{n}{(n+1)P' + P''Q} < 0. \tag{6}$$

We then total differentiate (3) and utilize (6) to obtain the comparative statics of a royalty on individual output as follows:

$$\frac{dq_i}{dr} = \frac{P' + P''(Q - nq_i)}{[(n+1)P' + P''Q]P'} < 0, \quad i = 1, 2, \cdots, n. \tag{7}$$

The results of (6) and (7) indicate that an increase in the royalty rate decreases the industry's total output, and that an increase in the royalty rate decreases the licensed firm's output. This is because a higher royalty raises the cost and thus reduces output

[19] The second-order condition is assumed to satisfy the requirement of a negative semi-definite matrix in which $2P' + P''q_i < 0$ and $[(n+1)P' + P''Q]P'^{n-1} > (<)\ 0$, for even (odd) n.

for the licensed firms.

Following the insights provided by Long and Soubeyran (1997), this asymmetric cost structure leads us to examine (7) in greater detail. With different costs, a royalty can influence firms in an asymmetric fashion. Let us denote the output of the low-cost (high-cost) firm as $q_i^L(q_i^H)$, where the superscripts L and H stand for low cost and high cost, respectively. Making use of (7) yields:

$$\frac{dq_i^L}{dr} - \frac{dq_i^H}{dr} = \frac{P''}{P'} \underbrace{(q_i^H - q_i^L)}_{-} \underbrace{\frac{dQ}{dr}}_{-} > (=, <)\ 0, \quad \text{if } P'' < (=, >)\ 0. \tag{8}$$

In (8), P'' is the curvature of market demand, whereby a negative (positive) magnitude displays concave (convex) market demand. Equation (8) shows that a higher royalty leads the low-cost firm to reduce less output than the high-cost firm does if the market demand is concave; on the contrary, if the market demand is convex, then the low-cost firm reduces more output than the high-cost firm does in response to a higher royalty. As for linear demand, the royalty affects both types of firms equivalently despite their costs being different. We write this result in Proposition 1.

Proposition 1 (demand-dependent production efficiency) An increase in royalty makes the high-cost (low-cost) firm reduce more output than the low-cost (high-cost) firm does when the market demand is concave (convex). Under linear demand, an increase in royalty has an equivalent influence among firms with different costs.

An increase in royalty decreases the output of the licensees, but the magnitude of the output decrease differs for the high-cost licensee and the low-cost licensee. The economic intuition behind Proposition 1 is straightforward. An increase in royalty improves (deteriorates) production efficiency for the industry when market demand is concave (convex), because the output decrease for the low-cost licensed firm is less (more) than that of the high-cost licensed firm in response to an increase in the royalty rate. We denote this as the effect of demand-dependent production efficiency in Proposition 1. Such an effect plays a crucial role in this paper. The result of Proposition 1 is similar in spirit to that of Long and Soubeyran (1997), who study optimal trade policies under the circumstance in which firms do not have identical unit costs, while this paper discusses the scenario associated with firms' licensing royalty.[20]

[20] On page 212 of Long and Soubeyran (1997), Proposition 3 says that a lower-cost firm's output will

We next study the industry's total profit, which is obtainable by adding up firms' profit after making use of (2) to (5) as follows:

$$\sum_{1}^{n} \Psi_i = -P' \sum_{1}^{n} q_i^2 - \sum_{1}^{n} F_i = -P' H Q^2 - \sum_{1}^{n} F_i. \tag{9}$$

Differentiating (9) with respect to Var(c_i) yields: [21]

$$\frac{d \sum_{1}^{n} \Psi_i}{d\text{Var}(c_i)} = \frac{-P' Q^2}{n(P - \overline{c}_n - r)^2} > 0. \tag{10}$$

Equation (10) shows that a higher market concentration increases the industry's total profit. The intuition behind (10) is as follows. Suppose the alternation of market concentration originates from the cost variation of firms. Given the sum of the industry's marginal costs, a higher cost variance is equivalent to a more concentrated market where low-cost firms increase output while high-cost firms decrease output. This leads to an increase (decrease) of market share for the low-cost (high-cost) firm and generates a higher degree of market concentration. Thus, a higher degree of market concentration that comes from an expanding cost variance not only lifts the industry's production efficiency, but also increases total profit for the whole industry. This production efficiency works under a different channel than Proposition 1. We coin this effect as variation-oriented production efficiency, in contrast to demand-dependent production efficiency, and write it as Proposition 2.

Proposition 2 (variation-oriented production efficiency) By taking licensing behavior into consideration, an increase in cost variation implies that lower-cost licensed firms increase output and higher-cost licensed firms decrease output; thus, the production efficiency and the total profit of the industry are enhanced.

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reduce more (less) than that of a higher-cost firm in response to a higher cost if and only if the demand curve is convex (concave).

[21] See Appendix 2.

3. MARKET CONCENTRATION AND LICENSING ROYALTY

The outsider patentee decides a licensing package to maximize its own profit from (1), and the licensees simultaneously accept or reject the offer in the first stage of the game. Based on conventional wisdom, a fixed fee F_i is charged up to the extent that the patentee can extract all the rents back, which means that F_i leaves the potential licensees indifferent between accepting or rejecting the offer. The outside option that a firm rejects the license is zero by Assumption 1. Accordingly, the patentee's objective function becomes:

$$\begin{aligned} \max \quad & \pi = rQ + \sum_{1}^{n} F_i, \\ \text{s.t.} \quad & \Psi_i = -P' q_i^2 - F_i \geq 0. \end{aligned} \tag{11}$$

Substituting the fixed fee back into patentee's profit and differentiating (11) with respect to the royalty rate yields the first-order condition for profit maximization as:

$$\frac{\partial \pi}{\partial r} = Q + r\frac{dQ}{dr} - P''\frac{dQ}{dr}\sum_{1}^{n} q_i^2 - 2P'\sum_{1}^{n} q_i \frac{dq_i}{dr} = 0. \tag{12}$$

Incorporating equations (5)–(7) into (12) and collecting terms, we obtain the optimal royalty as follows: [22]

$$r^* = \underbrace{P'Q\left(\frac{1}{n} - 1\right)}_{+} + P''Q^2 \underbrace{\left(\frac{1}{n} - H\right)}_{-}. \tag{13}$$

The asterisk in Equation (13) denotes the royalty rate being optimal. Equation (13) shows that under linear market demand, $r^* = P'Q(1-n)/n \geq 0$, which is not associated with the factor of market concentration; that is, market concentration is not involved in the determination of the royalty rate in linear demand. Moreover, $r^* = 0$

[22] Appendix 1 offers the derivation of (13).

if $n = 1$ in linear demand. In other words, in the case of one licensee, the patentee that is a profit maximizer at the top of the game with full bargaining power charges a zero royalty rate and a high fixed fee to grab the entire monopoly profit (see Sen and Tauman, 2007; Erutku et al., 2007; Wang et al., 2013).

Equation (13) shows two incentives for the patentee to adjust the royalty rate. The first incentive regards the licensing royalty directly collected from the licensees. The second incentive comes from the effect of demand-dependent production efficiency. It is intuitive to see from the first incentive that a higher r increases the licensed revenue yet decreases total output, which in turn decreases revenue for the patentee. As for the second incentive, a higher royalty improves production efficiency when demand is concave. This increased production efficiency allows the patentee to extract more rents back by means of the fixed fee. The above-mentioned incentives interact to determine the royalty.

A higher cost variation among firms results in a higher concentration index given industry quantity (see equations (4) and (5)). Accordingly, we derive the relationship between market concentration and licensing royalty by totally differentiating (12) as follows: [23]

$$\frac{dr^*}{d\text{Var}(c_i)} = -\frac{\pi_{r\ \text{var}}}{\pi_{rr}}, \tag{14}$$

where $\pi_{r\ \text{var}} = -P''Qn^2/\underbrace{[(n+1)P' + P''Q]P'}_{+} > (=,<)\ 0$ if $P'' < (=,>)\ 0$.

Note that the sign of equation (14) depends solely on the sign of $\pi_{r\ \text{var}}$. This result reveals a clear-cut link between market concentration and royalty – that is, the licensing royalty is independent of market concentration under linear market demand. However, if market demand is non-linear, then the licensing royalty closely corresponds to the market concentration. In other words, if market demand is concave (convex), then the nexus of the concentration index and royalty rate is positive (negative). We document this result in Proposition 3.

Proposition 3 The relationship between market concentration and licensing royalty depends on the curvature of market demand. If market demand is concave (convex)

[23] The second-order condition for the profit maximization presumably holds (i.e., $\pi_{rr} < 0$). Appendix 3 provides a specific example to show the relationship between market concentration and licensing royalty. We thank an anonymous referee for providing the suggestion for this example.

to the origin, then a higher market concentration is associated with a higher (lower) licensing royalty, whereas under linear demand, the licensing royalty is independent of market concentration.

Two forces come into play behind Proposition 3. The first force stated in Proposition 1(i), demand-dependent production efficiency, implies that low-cost firms decrease output less than high-cost firms do in reaction to a higher royalty if market demand is concave to the origin. This output adjustment among low-cost firms and high-cost firms in turn improves (deteriorates) production efficiency for the industry exhibiting concave (convex) market demand. Hence, the industry profit increases (decreases) under concave (convex) market demand, which allows the licensor to raise (lower) the royalty for extraction of the licensee's increased profit.

The second force originating from cost variance, variation-oriented production efficiency, further improves the industry's production efficiency, because a higher cost variation enables the lower-cost firms to increase output while the higher-cost firms decrease output. Due to this cost variation, the production efficiency effect not only raises market concentration, but also lifts the industry's profit level, which creates leeway for the licensor to charge a higher royalty. The demand-dependent production efficiency becomes a pivotal force in the relationship between market concentration and licensing royalty, in that higher market concentration results in a larger licensing royalty if market demand is concave. The case for convex demand works in the same spirit, but in the opposite direction. Finally, the demand-dependent production efficiency does not function in linear demand, and so market concentration has no effect on licensing royalty.[24]

In sum, the curvature of market demand plays a pivotal role in the relationship between the market concentration ratio and licensing royalty. To the best of our knowledge, this relationship has not yet been documented in the theoretical licensing literature on oligopoly.

4. DISCUSSION

We first explore the case where the patentee licenses the innovation in a discriminatory fashion and then discuss the case of exclusive licensing in linear demand.

[24] See Appendix 3 for the derivation of (12) in an example. The specific example that sets $k = 1$, $k = 2$, and $k = 1/2$ is provided upon request from the authors.

4.1 Discriminatory Two-Part Tariff

Even though the guiding rules of RAND or FRAND regulate the patentees to license the royalty rate in a non-discriminatory fashion, charging a royalty rate in a discriminatory fashion indeed occurs in practice.[25] In this section we proceed with an examination of the relationship between market concentration and the royalty rate under the circumstance of a discriminatory two-part tariff.

For expositional simplicity, the profit functions and the first-order conditions of profit maximization for the licensees are the same as (2) and (3), except that the royalty rate becomes firm specific, r_i. Appendix 4 shows the derivation of the comparative statics of the royalty on outputs, and the demand-dependent production efficiency is re-examined as follows:

$$\frac{dq_i}{dr_i} - \frac{dq_j}{dr_i} = \underbrace{\frac{1}{P'}}_{-} - \underbrace{\frac{P''(q_i - q_j)}{[(n+1)P' + P''Q]P'}}_{+} < 0, \quad i \neq j. \tag{15}$$

Equation (15) implies a regular impact in which the own effect of a higher r_i on the output of the licensed firm i is negative and the strategic substitution effect on the output of licensed firm j is positive. Similarly, we also obtain:

$$\frac{dq_j^L}{dr_i} - \frac{dq_j^H}{dr_i} = \underbrace{\frac{P''\overbrace{(q_j^H - q_j^L)}^{-}}{[(n+1)P' + P''Q]P'}}_{+} > (=, <)\, 0, \quad \text{if } P'' < (=, >)\, 0, \text{ for } i \neq j. \tag{16}$$

Equation (16) shows that the demand-dependent production efficiency effect is sustained at a discriminatory two-part tariff for the two groups of firms. The spirit behind (16) is the same as in Proposition 1. We list the above results as Proposition 4.

Proposition 4 For $i \neq j$, firm j with a high cost (low cost) reduces more output than a firm with a low cost (high cost) in response to a higher royalty r_i when market demand is concave (convex). For $i \neq j$, an increase in royalty r_i has an equivalent influence on

[25] We thank an anonymous referee for offering the firm-specific case of the royalty rate that helps enrich the analysis. The discussion on discriminatory tariff or pricing is not rare in the extant economics literature (see Choi, 1995; Swanson and Baumol, 2005; Chang et al., 2016).

firm j having different costs under linear demand.

The industry profit under a discriminatory two-part tariff is attainable by adding up firms' profit after making use of (A3e) and the first-order condition of (3) with the replacement of r_i. Utilizing $dH_{r_i}/d\text{Var}(c_i) = 1/(P - \overline{c}_n - \overline{r})^2$ in the industry profit, we can further calculate the link between cost variance and industry profit as $d\sum_1^n \Psi_i/d\text{Var}(c_i) = -P'Q^2/[n(P - \overline{c}_n - \overline{r})^2] > 0$. This $d\sum_1^n \Psi_i/d\text{Var}(c_i) > 0$ implies that an increase in cost variation gives rise to an increase in industry profit, and hence Proposition 2 holds under a discriminatory licensing contract.

We next re-examine the relationship between the royalty rate and market concentration in the setting of a discriminatory two-part tariff.[26] For expositional simplicity, we avoid repeating similar sentences or equations here. The profit function is given by $\pi = \sum_1^n r_i q_i + \sum_1^n F_i$. By Assumption 1, the first-order condition of profit maximization for the patentee is:

$$\frac{\partial \pi}{\partial r_i} = q_i + \sum_{j=1}^{n} r_j \frac{dq_j}{dr_i} - P'' \frac{dQ}{dr} \sum_{j=1}^{n} q_j^2 - 2P' \sum_{j=1}^{n} q_j \frac{dq_j}{dr_i} = 0, \quad i = 1, 2, \cdots, n. \tag{17}$$

We solve the n first-order conditions in (17) simultaneously to obtain the royalty rates as follows:[27]

$$r_i = -\frac{P}{e}\left(\frac{q_i}{Q} - 1\right) \geq 0, \tag{18}$$

where $e \equiv -(\partial Q/Q)/(\partial P/P)$ denotes the elasticity of demand function.

Equation (18) shows that the firm-specific royalty, r_i, depends on three factors: the demand function, the price elasticity of demand, and the market share of firm i. The larger the total output (quantity of firm i) is, the higher (smaller) the royalty rate will be. It is clear from (18) that the factor of market concentration does not play a part in determining the firm-specific royalty. Moreover, the royalty rate that the patentee charges is always non-negative. In particular, the royalty rate is zero in the case of one licensee, because the patentee can extract all the rents back from the monopoly by a fixed fee.

The case of the specific royalty rate shows some stark differences to Proposition 3,

[26] The re-examination on Proposition 2 is straightforward. We leave this part to Appendix 4(2).

[27] See the calculation of the specific royalty rate in Appendix 4(3).

whereby the market concentration is irrelevant to the determination of the firm-specific royalty rate. We write the above observation as Proposition 5.

Proposition 5 The optimal firm-specific royalty rate is inversely related to individual output in which a higher output results in a lower royalty rate. Moreover, the market concentration ratio is irrelevant for the determination of a firm-specific royalty rate.

4.2 Exclusive Licensing

This subsection shows the case where 2 potential licensees are present and have costs $c+\delta$ and $c-\delta$ under the demand of $P = q-Q$, in order to examine the patentee's choice in exclusive licensing. Note that δ^2 indicates the cost variance of firms. When the patentee is confined to exclusive licensing, the optimal royalty rate is zero, irrespective of the licensee being the low-cost firm or the high-cost one (see Appendix 5). However, the patentee's profit is different with respect to the licensee, in that the patentee gains higher profit by licensing to the low-cost firm than to the high-cost firm. The reason is intuitive, because even though the patentee charges a zero royalty rate to the licensee, it can still capture the entire monopoly profit from the licensee via the fixed fee. The lower the cost to the licensee is, the more profit the patentee can capture. It is a well-known result in the licensing literature that a patentee licenses to the lower-cost firm instead of the higher-cost firm under exclusive licensing, which is consistent with Sen and Tauman (2007), Erutku et al. (2007), and Wang et al. (2013).

We learn that the patentee is motivated to license to the lower-cost firm in exclusive licensing, and the profit is higher with exclusive licensing than with non-exclusive licensing (see Appendix 5). The inquiry as to why it is common to observe non-exclusive licensing turns out to be quite interesting. Let us check this from the perspective of global welfare, which consists of patentee profit, licensees' profits, and consumer surplus. We see that the cost variation, δ^2, is a factor that is deterministic to the relative magnitude of welfare for exclusive or non-exclusive licensing. The welfare is the same either in non-exclusive licensing or exclusive licensing when $\delta^2 = 0$, implying that non-exclusive licensing is no worse than exclusive licensing when firms' costs are similar. On the other hand, the welfare is smaller (larger) in non-exclusive licensing than exclusive licensing to the lower- (higher-) cost firm when $\delta^2 > 0$. As Wang et al. (2013) point out, the factor of relative cost between firms plays an important role in the determination of exclusive or non-exclusive licensing. However, the cost structure or complete information of a firm's cost structure is usually unobservable to governments and businesses (e.g., Gresik, 2001; Nielsen et al., 2003), such that the law of non-exclusive licensing highlighted in RAND might be the second-best policy to the public.[28]

5. CONCLUDING REMARKS

While numerous works have generated penetrating insights into the issue of patent licensing over the last decade, to the best of our knowledge, the nexus between licensing royalty and market concentration has not yet been investigated in the theoretical licensing literature. To fill this theoretical gap, we analyze the correlation of licensing royalty and market concentration index under an asymmetric oligopoly. This paper further interchangeably discusses the relationships among market concentration, licensing royalty, and non-linear market demand by introducing non-linear demand and asymmetric costs into the licensing framework. One assumption is introduced to facilitate the analysis, that is, a firm joins the production stage as long as it accepts the technology licensing. We show that the curvature of market demand has a decisive influence on the nexus of licensing royalty and concentration index. When market demand is linear, the market concentration is independent of the royalty rate. However, the relationship between market concentration and royalty rate is positive (negative) when market demand is concave (convex) to the origin.

Despite the clear-cut relationship between market concentration and licensing royalty, readers should note that this paper employs cost variance as a proxy for market concentration to conduct the analysis throughout the whole paper. It is the cost variance, Var(c_i), that affects these two variables. Moreover, this paper does not shed light on the optimal licensing strategy for the patentee – i.e., whether the optimal licensing strategy is a fixed fee, a royalty, or a two-part tariff. Therefore, the results derived herein are more applicable to scenarios associated with a royalty. Nonetheless, the information obtained in this paper can benefit regulators that are concerned about the status of market concentration and intend to create a sound industrial environment. The findings of this paper also provide a reference from the aspect of licensing royalty for patentees that offer a licensing royalty contingent upon market concentration to

[28] Both standard and essential patents often bear an obligation for licensing on reasonable and non-discriminatory terms, although the "non-discriminatory" requirement of the RAND obligation is still not fully settled (Boger and Frankel, 2014).

licensees following FRAND terms. More extensions can be incorporated into future study in order to examine the relationship between market concentration and royalty, such as the trait of horizontal product differentiation, the possibility of free entry, or the introduction of a tax or subsidy on a firm's innovation. These are interesting topics for future research.

APPENDIX 1

Appendix 1 first shows the derivation of the Herfindahl-Hirschman Index and then derives equation (13).

(1) By definition, this index is the square sum of each firm's market share. After making use of (3), we obtain:

$$H = \frac{\sum_1^n q_i^2}{Q^2} = \frac{\sum_1^n (P - c_i - r)^2}{P'^2 Q^2}. \tag{A1a}$$

Utilizing (4) into (A1a) yields:

$$H = \frac{\sum_1^n (P - \overline{c}_n - r + \overline{c}_n - c_i)^2}{[n(P - \overline{c}_n - r)]^2} = \frac{\mathrm{Var}(c_i) + (P - \overline{c}_n - r)^2}{n(P - \overline{c}_n - r)^2},$$

(2) The first-order condition for profit maximization of (12) is copied as follows:

$$\frac{\partial \pi}{\partial r} = Q + r\frac{dQ}{dr} - P''\frac{dQ}{dr}\sum_1^n q_i^2 - 2P'\sum_1^n q_i \frac{dq_i}{dr} = 0. \tag{A1b}$$

We set (A1b) to zero, rearrange terms before utilizing (5) and (6) into (A1b), and collect terms that yield the following:

$$\begin{aligned} r &= \frac{\left(P''\frac{dQ}{dr}\sum_1^n q_i^2 + 2P'\sum_1^n q_i\frac{dq_i}{dr} - Q\right)}{\frac{dQ}{dr}} \\ &= P''\sum_1^n q_i^2 + \frac{2\left[P'Q + P''\sum_1^n (Q - nq_i)q_i\right]}{n} - Q[(n+1)P' + P''Q] \\ &= \frac{P'Q(1-n)}{n} + P''\left(\frac{Q^2}{n} - \sum_1^n q_i^2\right). \end{aligned}$$

APPENDIX 2

Appendix 2 derives equation (10). Note that P is related to the total quantity and is irrelevant to the cost variance as long as the sum holds. We derive the square sum of the firms as $\sum_1^n q_i^2 = \left[\sum_1^n (P - \bar{c}_n - r)^2 + n\text{Var}(c_i)\right] / P'^2$. Making use of the information and then differentiating (9) with respect to cost variation yield:

$$\frac{d\sum_1^n \Psi_i}{d\text{Var}(c_i)} = \frac{-P'Q^2}{n(P - \bar{c}_n - r)^2} > 0.$$

APPENDIX 3

Appendix 3 shows (1) the relationship between market concentration and licensing royalty under the demand of $P = a - Q^k$ and (2) the relationship between market concentration and licensing royalty under the demand of $P = a - Q^k$, where $n = 2$, and the costs of the two licensees are $c + \delta$ and $c - \delta$. Here, δ^2 in this example indicates the cost variance of the two firms.

(1) Recall that the market demand is convex (linear, concave) to the origin if $k < (=, >)$ 1. We solve backwards to obtain the last-stage equilibrium as follows:

$$Q^k = \frac{na - \sum_1^n c_i - nr}{n + k}. \tag{A2a}$$

Applying Cramer's Rule, the comparative statics are derivable as follows:

$$\frac{dQ}{dr} = \frac{-nQ^{1-k}}{(n + k)k}. \tag{A2b}$$

Utilizing the information from (A2a) to (A2b) into (12), we can derive the first-order condition for the patentee's profit maximization as follows:

$$\frac{\partial \pi}{\partial r} = Q + r[-n-(k-1)]\frac{dQ}{dr} - \frac{nr}{kQ^{k-1}} + 2kQ^{k-1}\sum_1^n q_i$$
$$\left[-\frac{dQ}{dr} - (k-1)\frac{q_i}{Q}\frac{dQ}{dr} - \frac{1}{kQ^{k-1}}\right] + k(k-1)Q^{k-2}\frac{dQ}{dr}\sum_1^n q_i^2 = 0. \tag{A2c}$$

We now totally differentiate (A2c) to obtain (14). The numerator of (14) is as follows:

$$\pi_{r\text{ var}} = \frac{k(1-k)}{Q^2}\underbrace{\frac{dQ}{dr}}_{-}\underbrace{\left[\frac{n}{kQ^{k-1}} + n\left(\frac{\sum_1^n q_i}{n}\right)^2\right]}_{+} > (=,<)\ 0, \quad \text{if } k > (=,<)\ 1. \tag{A2d}$$

With (A2d), we can tell that a higher market concentration leads to a higher (lower) royalty when market demand is concave (convex) by (14).

(2) With the demand of $P = a - Q^k$, $n = 2$, the costs to firms are $c + \delta$ and $c - \delta$. Note that the industry output is $Q = q_1 + q_2$. The quantity and the comparative statics of a royalty on quantity are shown as follows:

$$Q^k = \frac{2(a-c-r)}{2+k},\ q_1 = \frac{(a-c-r-\delta-Q^k)}{kQ^{k-1}},\ q_2 = \frac{(a-c-r+\delta-Q^k)}{kQ^{k-1}}, \tag{A2e}$$

$$\frac{dQ}{dr} = \frac{-2Q^{1-k}}{(2+k)k},\ \frac{dq_1}{dr} = \frac{(1-k)q_1}{Q} - \frac{Q^{1-k}}{k} - \frac{dQ}{dr},$$
$$\frac{dq_2}{dr} = \frac{(1-k)q_2}{Q} - \frac{Q^{1-k}}{k} - \frac{dQ}{dr}. \tag{A2f}$$

Substituting (A2e) and (A2f) into (12), we obtain the first-order condition for the optimal royalty as follows:

$$\frac{\partial \pi}{\partial r} = \frac{Q}{2+k} - \frac{2rQ^{1-k}}{(2+k)k} + \frac{4(k-1)\delta^2 Q^{1-2k}}{k^2(2+k)} = 0. \tag{A2g}$$

Further differentiating (A2g) with respect to δ^2 yields:

$$\frac{\partial\left(\dfrac{\partial \pi}{\partial r}\right)}{\partial \delta^2} = \frac{4(k-1)Q^{1-2k}}{k^2(2+k)} > (<,=)\ 0, \quad \text{if } k > (<,=)\ 0. \tag{A2h}$$

Here, (A2h) shows that a greater cost variance leads to a larger (smaller) royalty if demand is concave (convex) to the origin.

APPENDIX 4

The purpose of Appendix 4 is threefold. Appendix 4 first derives the comparative statics of the firm-specific royalty rates on outputs, then the concentration index under the case of a firm-specific royalty, and lastly the optimal royalty firm-specific rates for the patentee.

(1) Assume the second-order condition satisfies the negative semi-definite requirement. We replace the royalty rate in Eqs. (1) to (3) with the firm-specific royalty rate before summing up all the first-order conditions to obtain the comparative static of r_i on total output as follows:

$$\frac{dQ}{dr_i} = \frac{1}{(n+1)P' + P''Q} < 0, \quad i = 1, 2, \cdots, n. \tag{A3a}$$

We then differentiate equation (3) and utilize (A3a) to obtain the comparative statics of r_i on individual output q_i and q_j as follows:

$$\frac{dq_i}{dr_i} = \frac{nP' + P''(Q-q_i)}{[(n+1)P' + P''Q]P'} < 0, \quad i = 1, 2, \cdots, n. \tag{A3b}$$

$$\frac{dq_j}{dr_i} = \frac{-(P' + P''q_j)}{[(n+1)P' + P''Q]P'} > 0, \quad i \neq j. \tag{A3c}$$

(2) Following the process in Appendix 1, we obtain H as follows:

$$H_{r_i} = \frac{\sum_1^n q_i^2}{Q^2} = \frac{\sum_1^n (P - c_i - r_i)^2}{P'^2 Q^2}, \tag{A3d}$$

where the subscript r_i represents the case of a firm-specific royalty rate.

Subtracting the mean of the costs and royalty rates before making use of (3) into (A3d) yields:

$$\begin{aligned} H_{r_i} &= \frac{\sum_1^n (P - \overline{c}_n - \overline{r} + \overline{r} - r_i + \overline{c}_n - c_i)^2}{[n(P - \overline{c}_n - \overline{r})]^2} \\ &= \frac{\text{Var}(c_i) + \text{Var}(r_i) + (P - \overline{c}_n - \overline{r})^2}{n(P - \overline{c}_n - \overline{r})^2}, \end{aligned} \tag{A3e}$$

where $\overline{r} \equiv \sum_1^n r_i / n$.

(3) List the first-order conditions in the matrix as follows:

$$\begin{bmatrix} \frac{dq_1}{dr_1} & \frac{dq_2}{dr_1} & \cdots & \frac{dq_n}{dr_1} \\ \frac{dq_1}{dr_2} & \frac{dq_2}{dr_2} & \cdots & \frac{dq_n}{dr_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{dq_1}{dr_n} & \frac{dq_2}{dr_n} & \cdots & \frac{dq_n}{dr_n} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = \begin{bmatrix} 2P' \sum_1^n q_i \frac{dq_i}{dr_1} + P'' \frac{dQ}{dr_1} \sum_1^n q_i^2 - q_1 \\ 2P' \sum_1^n q_i \frac{dq_i}{dr_2} + P'' \frac{dQ}{dr_2} \sum_1^n q_i^2 - q_2 \\ \vdots \\ 2P' \sum_1^n q_i \frac{dq_i}{dr_n} + P'' \frac{dQ}{dr_n} \sum_1^n q_i^2 - q_n \end{bmatrix}.$$

By Cramer's rule, we solve r_1 as follows:

$$r_1 = \frac{\Delta_1}{\Delta}. \tag{A3f}$$

where Δ is the determinant of the $n \times n$ matrix to the left-hand side of (A3f), and Δ_1 is the determinant of the $n \times n$ matrix after replacing the first column with the vector in the right-hand side of (A3f).

The property of the determinant says that row operations or column operations do not alter the determinant. After two rounds of operations, we make use of the Laplace expansion to the determinant of the $n \times n$ matrix, which yields:

$$
\begin{aligned}
\Delta &= \begin{vmatrix} \dfrac{dq_1}{dr_1} & \dfrac{dq_2}{dr_1} & \cdots & \dfrac{dq_n}{dr_1} \\ \dfrac{dq_1}{dr_2} & \dfrac{dq_2}{dr_2} & \cdots & \dfrac{dq_n}{dr_2} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{dq_1}{dr_n} & \dfrac{dq_2}{dr_n} & \cdots & \dfrac{dq_n}{dr_n} \end{vmatrix} \\
&= \begin{vmatrix} \dfrac{nP' + P''(Q-q_1)}{P'[(n+1)P' + P''Q]} & \dfrac{-(P' + P''q_2)}{P'[(n+1)P' + P''Q]} & \cdots & \dfrac{-(P' + P''q_n)}{P'[(n+1)P' + P''Q]} \\ \dfrac{-(P' + P''q_1)}{P'[(n+1)P' + P''Q]} & \dfrac{nP' + P''(Q-q_2)}{P'[(n+1)P' + P''Q]} & \cdots & \dfrac{-(P' + P''q_n)}{P'[(n+1)P' + P''Q]} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{-(P' + P''q_1)}{P'[(n+1)P' + P''Q]} & \dfrac{-(P' + P''q_2)}{P'[(n+1)P' + P''Q]} & \cdots & \dfrac{nP' + P''(Q-q_n)}{P'[(n+1)P' + P''Q]} \end{vmatrix} \\
&= \frac{1}{P'^{n-1}[(n+1)P' + P''Q]}.
\end{aligned}
$$

By employing the row operations, column operations, and Laplace expansion similar to the above calculation, we obtain the following:

$$
\begin{aligned}
\Delta_1 &= \begin{vmatrix} 2P'\sum_1^n q_i \dfrac{dq_i}{dr_1} + P''\dfrac{dQ}{dr_1}\sum_1^n q_i^2 - q_1 & \dfrac{dq_2}{dr_1} & \cdots & \dfrac{dq_n}{dr_1} \\ 2P'\sum_1^n q_i \dfrac{dq_i}{dr_2} + P''\dfrac{dQ}{dr_2}\sum_1^n q_i^2 - q_2 & \dfrac{dq_2}{dr_2} & \cdots & \dfrac{dq_n}{dr_2} \\ \vdots & \vdots & \ddots & \vdots \\ 2P'\sum_1^n q_i \dfrac{dq_i}{dr_n} + P''\dfrac{dQ}{dr_n}\sum_1^n q_i^2 - q_n & \dfrac{dq_2}{dr_n} & \cdots & \dfrac{dq_n}{dr_n} \end{vmatrix} \\
&= \begin{vmatrix} q_1 - \dfrac{2P'Q + P''\sum_1^n q_i^2}{(n+1)P' + P''Q} & \dfrac{-(P' + P''q_2)}{[(n+1)P' + P''Q]P'} & \dfrac{-(P' + P''q_3)}{[(n+1)P' + P''Q]P'} & \cdots & \dfrac{-(P' + P''q_n)}{[(n+1)P' + P''Q]P'} \\ q_2 - q_1 & \dfrac{1}{P'} & 0 & \cdots & 0 \\ q_3 - q_1 & 0 & \dfrac{1}{P'} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_n - q_1 & 0 & 0 & \cdots & \dfrac{1}{P'} \end{vmatrix} \\
&= \frac{1}{P'^{n-1}} \frac{P'(q_1 - Q)}{(n+1)P' + P''Q}.
\end{aligned}
$$

Finally, substituting Δ and Δ_1 back into (A3f) yields $r_1 = P'(q_1 - Q)$.

APPENDIX 5

This appendix shows the profit for the patentee under exclusive licensing and the associated welfare for the circumstances in which two potential licensees, firms 1 and 2, have costs of $c+\delta$ and $c-\delta$, and the demand is $P = a-Q$. The global welfare function consists of the profits of the licensees and the patentee, and the consumer surplus.

(1) The profit functions for the firms if both firms are licensed are $\Psi_1 = (a - Q - (c + \delta) - r)q_1 - F_1$ and $\Psi_2 = (a - Q - (c - \delta) - r)q_2 - F_2$, respectively. If a firm is not licensed, then it hardly survives in the business and receives zero profit according to Assumption 1. The patentee's objective function when licensing to firm i is to maximize $\pi = rq_i + F_i$, such that $\Psi_i > 0$.

If the patentee licenses to firm 1 only, then the second-stage equilibrium is $q_1 = (a - c - \delta - r)/2$ and a unit increase in royalty gives rise to 0.5 units of decreased output. Next, we substitute the output into the patentee's profit to obtain the first-order condition for the determination of the royalty rate as follows:

$$q_1 + r\frac{dq_1}{dr} + 2q_1\frac{dq_1}{dr} = 0. \tag{A4}$$

The royalty rate obtained from (A4) is zero. The patentee's profit is obtainable by substituting the zero royalty rate and the fixed fee back to π. Alternatively, if the patentee licenses to firm 2 only, then by taking similar procedures, we obtain the optimal royalty rate of $r = 0$. Let us show the patentee's profit in the two cases.

$$\pi = \begin{cases} \dfrac{[a-(c+\delta)]^2}{4}, & \text{if the high-cost firm takes exclusive licensing,} \\ \dfrac{[a-(c-\delta)]^2}{4}, & \text{if the low-cost firm takes exclusive licensing.} \end{cases}$$

It is obvious from the above that the patentee acquires a higher profit after licensing to the firm with a lower cost under exclusive licensing.

(2) The welfare associated with exclusive licensing is higher with the lower-cost firm than with the higher-cost firm since the welfare with the former is $3[a-(c-\delta)]^2/8$ and

the welfare with the latter is $3[a-(c+\delta)]^2/8$. For non-exclusive licensing whereby the two firms are both licensed, the optimal royalty rate is $r=(a-c)/4$. The outputs of the two firms are $q_1=(a-c)/4-\delta$ and $q_2=(a-c)/4+\delta$. The patentee's profit is $(a-c)^2/4+2\delta^2$, and the associated welfare is $3(a-c)^2/8+2\delta^2$.

REFERENCES

Acs, Z. J. and D. B. Audretsch (1988), "Innovation in Small and Large Firms: An Empirical Analysis," *American Economic Review*, 78, 678–690.

Aguirre, I., S. Cowan, and J. Vickers (2010), "Monopoly Price Discrimination and Demand Curvature," *American Economic Review*, 100, 1601–1615.

Battersby, G. J. and C. W. Grimes (2015), *Licensing Royalty Rates: 2016 Edition*, New York: Wolters Kluwer Law & Business.

Bergstrom, T. and H. Varian (1985), "When Are Nash Equilibria Independent of the Distribution of Agents' Characteristics?" *Review of Economic Studies*, 52, 715–718.

Boger, A. S. and K. M. Frankel (2014), "What Does 'Non-Discriminatory' Mean in RAND Licensing of Standard-Essential Patents?" *LES Insights*, June 16.

Brander, J. and B. Spencer (1985), "Export Subsidies and International Market Share Rivalry," *Journal of International Economics*, 18, 83–100.

Chang, R.-Y., H. Hwang, and C.-H. Peng (2013), "Technology Licensing in Multiple Markets," *Academia Economic Papers*, 41, 597–613.

Chang, R.-Y., H. Hwang, and C.-H. Peng (2016), "Discriminatory vs. Uniform Tariffs with International Technology Licensing," *Asia-Pacific Journal of Accounting & Economics*, 23, 268–277.

Choi, J. P. (1995), "Optimal Tariffs and the Choice of Technology Discriminatory Tariffs vs. the 'Most Favored Nation' Clause," *Journal of International Economics*, 38, 143–160.

Collie, D. (1993), "Strategic Trade Policy under Asymmetric Oligopoly," *European Journal of Political Economy*, 9, 275–280.

Erutku, C., A. Priegue Freire, and Y. Richelle (2007), "Licensing Innovations with Exclusive Contracts," *Review of Industry Organization*, 31, 261–273.

Eswaran, M. (1994a), "Licensees as Entry Barriers," *Canadian Journal of Economics*, 27, 673–688.

Eswaran, M. (1994b), "Cross-Licensing of Competing Patents as a Facilitating Device," *Canadian Journal of Economics*, 27, 689–708.

Fauli-Oller, R. and J. Sandonis (2002), "Welfare Reducing Licensing," *Games and Economic*

Behavior, 41, 192–205.

Fosfuri, A. and E. Roca (2004), "Optimal Licensing Strategy: Royalty or Fixed-Fee?" *International Journal of Business and Economics*, 3, 13–19.

Gresik, T. (2001), "The Taxing Task of Taxing Transnationals," *Journal of Economic Literature*, 39, 800–838.

Ishida, J., T. Matsumura, and N. Matsushima (2011), "Market Competition, R&D and Firm Profits in Asymmetric Oligopoly," *Journal of Industrial Economics*, 59, 484–505.

Kamien, M. (1992), "Patent Licensing," in R. J. Aumann and S. Hart (eds.), *Handbook of Game Theory with Economic Applications*, 331–354, Amsterdam; New York: North-Holland.

Kamien, M. and Y. Tauman (1984), "The Private Value of a Patent: A Game Theoretic Analysis," *Journal of Economics*, Supplement 4, 93–118.

Kamien, M. and Y. Tauman (1986), "Fees versus Royalties and the Private Value of a Patent," *Quarterly Journal of Economics*, 101, 471–491.

Katz, M. and C. Shapiro (1986), "How to License Intangible Property," *Quarterly Journal of Economics*, 101, 567–589.

Lee, S. H. and S. Quayes (2005), "When Is Licensing Better Facilitated? Global Evidence from the Telecom Sector," *Global Economic Review: Perspectives on East Asian Economies and Industries*, 34, 119–127.

Lin, L. and N. Kulatilaka (2006), "Network Effects and Technology Licensing with Fixed Fee, Royalty, and Hybrid Contracts," *Journal of Management Information Systems*, 23, 91–118.

Long, N. V. and A. Soubeyran (1997), "Cost Heterogeneity, Industry Concentration and Strategic Trade Policies," *Journal of International Economics*, 43, 207–220.

Melitz, M. (2003), "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Mukherjee, A. and S. Mukherjee (2005), "Entry, Licensing and Welfare," Discussion Paper, University of Nottingham.

Nadiri, M. I. (1993), "Innovations and Technological Spillovers," *NBER Working Paper*, No. 4423.

Neary, J. P. (1994), "Cost Asymmetries in International Subsidy Games: Should Government Help Winners or Losers?" *Journal of International Economics*, 37, 197–218.

Nielsen, S., P. Raimondos-Moller, and G. Schjelderup (2003), "Formula Apportionment and Transfer Pricing under Oligopolistic Competition," *Journal of Public Economic Theory*, 5, 417–436.

Norback, P. and L. Persson (2004), "Licensing and Foreign Competition," *Journal of International Economics*, 62, 409–416.

Park, M., S. W. Lee, and Y. J. Choi (2011), "Does Spectrum Auctioning Harm Consumers? Lessons from 3G Licensing," *Information Economics and Policy*, 23, 118–126.

Peria, M. S. M. and A. Mody (2004), "How Foreign Participation and Market Concentration Impact Bank Spreads: Evidence from Latin America," *Journal of Money, Credit and Banking*, 36, 511–537.

Poddar, S. and U. B. Sinha (2008), "On Patent Licensing," in R. Cellini and L. Lambertini (eds.), *The Economics of Innovation*, 33–65, Bingley: Emerald Group Publishing Limited.

Poddar, S. and U. B. Sinha (2010), "Patent Licensing from a High-Cost Firm to a Low-Cost Firm," *Economic Record*, 86, 384–395.

Rockett, K. (1990), "The Quality of Licensed Technology," *International Journal of Industrial Organization*, 8, 559–574.

Rostocker, M. (1984), "A Survey of Corporate Licensing," *IDEA: The Journal of Law and Technology*, 24, 59–92.

Sen, D. and Y. Tauman (2007), "General Licensing Schemes for a Cost-Reducing Innovation," *Games and Economic Behavior*, 59, 163–186.

Silva, F., M. J. Braga, and J. C. Garcia (2015), "Link between R&D Intensity and Market Concentration: Analysis of Brazilian Corn and Soybean Seed Markets," Conference Paper, the 29th International Conference of Agricultural Economists.

Stavins, J. (2001), "Price Discrimination in the Airline Market: The Effect of Market Concentration," *Review of Economics and Statistics*, 83, 200–202.

Swanson, D. G. and W. J. Baumol (2005), "Reasonable and Nondiscriminatory (RAND) Royalties, Standards Selection, and Control of Market Power," *Antitrust Law Journal*, 73, 1–58.

Vernon, R. (1979), "The Product Cycle Hypothesis in a New International Environment," *Oxford Bulletin of Economics and Statistics*, 41, 255–267.

Villalonga, B. (2000), "Licensing and Efficiency: Differentiating Ownership Effects from Political, Organizational, and Dynamic Effects," *Journal of Economic Behavior & Organization*, 42, 43–74.

Vishwasrao, S. (2007), "Royalties vs. Fees: How Do Firms Pay for Foreign Technology?" *International Journal of Industrial Organization*, 25, 741–759.

Wang, K. C. A., W. J. Liang, and P. S. Chou (2013), "Patent Licensing under Cost Asymmetry among Firms," *Economic Modelling*, 31, 297–307.

Yang, C. H. and M. W. Tsou (2016), "Technological Regimes, R&D Agglomeration and R&D: Does Firm Size Matter?" *Academia Economic Papers*, 44, 35–62.

異質寡占下之市場集中度與單位授權金

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摘　要

本文以一個非對稱的寡占模型來探討市場集中度與單位授權金的關係, 在模型中, 技術母廠遵循RAND原則以單位授權金及固定權利金的組合授權給在市場競爭的廠商。以成本變異作為市場集中度的代理變數下, 本文研究表明當市場需求為線性時, 單位授權金與市場集中度是獨立的; 但是, 當市場需求為凹向原點時, 市場集中度與單位授權金呈現正向關係, 即當市場集中度愈高時, 單位授權金也會提升, 但是此一正向關係在市場需求為凸向原點時會轉變成負向關係。因此, 消費市場的需求情形對於單位授權金與市場集中度兩者之間的關係具有關鍵性影響。