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On an EOQ Model with Ramp Type Demand Rate and Time Dependent Deterioration Rate

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Abstract

Recently, Manna and Chaudhuri [9] studied the EOQ model with ramp type demand rate, time-dependent deterioration rate, unit production cost and shortages. In this article, we extend their model by considering the backlogging rate, which is a negative exponential function of the waiting time until the next replenishment. In addition, we also add the setup cost into the model, and eliminate half of the unnecessary decision variables. We then use the commercial computer software, Mathematica, to find the optimal production policy.

Keywords: Inventory, Deteriorating Items, Partial Backlogging, Ramp Type Function.

1. Introduction

In inventory management, many researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic components and fashion goods. Ghare and Schrader [5] were the first proponents for developing a model for an exponentially decaying inventory. They categorized decaying inventory into three types: direct spoilage, physical depletion and deterioration. Next, Misra [11] studied a Weibull deterioration rate for the perishable product without allowing for backlogging of demand. Dave and Patel [4] considered an EOQ model for deteriorating items with time-proportional demand when shortages were prohibited. Sachan [12] then generalized

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the EOQ model to allow for shortages. Later, Hariga [7] generalized the demand pattern to any log-concave function. Teng et al. [13] and Yang et al. [18] further generalized the demand function to include any non-negative, continuous function that fluctuates with time.

Balkhi and Benkherouf [3] developed a general EPQ model for deteriorating items where demand and production rates are time varying, but the rate of deterioration is constant. Balkhi [2] then further generalized the EPQ model to allow for time-varying deterioration rate. Concurrently, Yan and Cheng [17] considered a perishable single-item EPQ model in which production rate, demand rate and deterioration rate are assumed to be functions of time, and shortages are partially backlogged.

Recently, Manna and Chaudhuri [9] investigated the EOQ model for time-dependent deteriorating items, and assumed the demand rate to be a ramp type function of time and the unit production cost to be inversely proportional to the demand rate. This type of demand pattern can be widely seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time(in the present model, we have assumed a linear trend) up to a certain time, μ, and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic.(e.g. see Hill [8], Mandal and Pal [10], Wu et al. [14], Wu and Ouyang [16], Wu [15] and Giri et al. [6]). However, In Manna and Chaudhuri [9], they did not consider the setup cost into their model so that they could underestimate their total average cost. In addition, they assumed that the production run time (t_1) is independent of the time with no production (t_2) in the model without shortage. In fact, the time with no production is a function of the production run time. Consequently, they used two decision variables, instead of a single decision variable. Similarly, they had too many decision variables for the case of shortages. Meanwhile, they mislaid the production cost and deterioration cost into their total average cost simultaneously. However, the production cost consists of the deterioration cost and undeterioration cost. In this article, we not only amend those shortcomings but also extend their fully backlogging assumption to the case of partial backlogging.

2. Notation and Assumptions

The following notation, most are similar to those in Manna and Chaudhuri [9] model.

A	setup cost per setup.
R	the demand rate, known function of time t.
K	the production rate, known function of time t.
v	unit production cost is equal to $\alpha_1 R^{-\gamma}$, where $\alpha_1 > 0$ and $2 > \gamma > 0$.
c_1	unit holding cost per unit time.
c_2	unit shortage cost per unit time for backlogged items.
c_3	unit cost of lost sales.
t_1	time at which the inventory level is at maximum.
t_2	inventory cycle time when the model without shortage (i.e., Model 1), or time at which shortages start to occur when the model with shortage (i.e., Model 2).
t_3	the production restarting time when the model with shortage (i.e., Model 2).
t_4	inventory cycle time when the model with shortage (i.e., Model 2).
$I_i(t)$	the on-hand stock level if it is positive; otherwise, it is the number of backorders at time t in Phase i, $i = 1, 2, 3, 4$ and 5.
TAC_1	the total cost per unit time of the system in Model 1.
TAC_2	the total cost per unit time of the system in Model 2.
TAC_1^*	the optimum total cost per unit time of the system in Model 1.
TAC_2^*	the optimum total cost per unit time of the system in Model 2.

In addition, the following assumptions are used throughout this paper.

(1) The planning horizon is infinite.

(2) Lead time is zero.

(3) The initial and final inventory levels are both zero.

(4) In the Model 2, the shortages are allowed. However, the longer the waiting time, the smaller the backlogging rate. Here, we assumed the backlogging rate, $b(\tau)$, is a negative exponential function of the waiting time up to the next replenishment τ, i.e., $b(\tau) = k_0 e^{-k_1 \tau}$, $k_0 \leq 1$, $0 \leq k_1$. (For example, see Abad [1]).

(5) Demand rate $R = f(t)$ is assumed to be a ramp type function of time. That is, $f(t) = D_0[t - (t-\mu)H(t-\mu)]$, where D_0 and μ are positive constants (see Figure 1), and $H(t-\mu)$ is the Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & \text{if } t \geq \mu, \\ 0 & \text{if } t < \mu. \end{cases}$$

(6) $K = \beta f(t)$ is the production rate, where $\beta(> 1)$ is a constant.

(7) A variable fraction $\theta(t) = \alpha t$ ($0 < \alpha << 1$ and $t \geq 0$) is the deterioration rate. During the early stage of inventory, the intensity of deterioration is very low because t is small. However, the intensity increases with time, but $\theta(t)$ remains bounded for $t >> 1$ since $0 < \alpha << 1$.

(8) There is no repair or replacement of the deteriorated inventory. Hence, there is no salvage value for the deteriorated items.

(9) The production run time (t_1) in without shortage period is greater than μ.

In this paper, we assume that the vendor's objective is to minimize the total cost per unit time of the system.

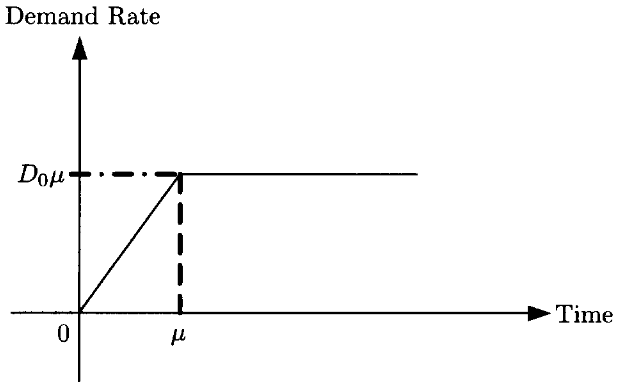

Figure 1. The ramp type demand rate.

3. Mathematical Formulations and Theoretical Results

In this section, we will discuss two models: one is without shortage, and the other is with partial backlogging.

Model 1. Model without shortage

In this model, the production starts with zero stock level at time $t = 0$ and the production stops at time t_1. Due to the combined effects of demand and deterioration of items, the inventory level gradually diminishes during the period $[t_1, t_2]$ and ultimately falls to zero at time $t = t_2$. The whole process is repeated and the behaviour of the inventory system is depicted in Figure 2. We know from Figure 2 that the inventory cycle here has the following three phases:

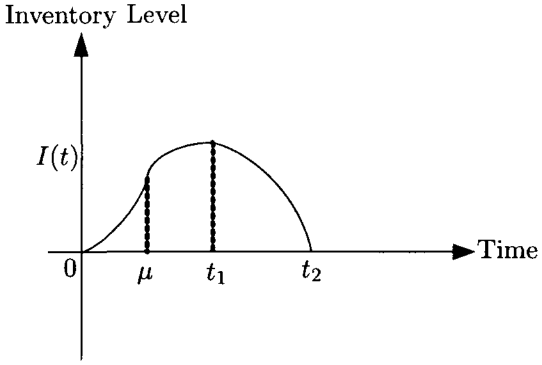

Figure 2. The inventory model without shortage.

Phase 1. During the time interval $[0,\mu]$, the demand rate is D_0t, the production rate is $\beta D_0 t$, and the deterioration rate is $\alpha t I_1(t)$ at time t. Therefore, the inventory level at time t, is governed by

$$\frac{dI_1(t)}{dt}+\alpha t I_1(t)=(\beta-1)D_0 t,\quad 0\le t\le \mu, \tag{1}$$

with the boundary condition. $I_1(0)=0$.

Phase 2. During the time interval $[\mu,t_1]$, from assumptions (5)-(7) and Figures 1-2, we know that the demand rate is $D_0\mu$, the production rate is $\beta D_0\mu$, and the deterioration rate is $\alpha t I_2(t)$ at time t. Therefore, the inventory level at time t, is governed by

$$\frac{dI_2(t)}{dt}+\alpha t I_2(t)=(\beta-1)D_0\mu,\quad \mu\le t\le t_1. \tag{2}$$

Phase 3. In the time interval $[t_1,t_2]$, the system is affected by the combined the demand and deterioration. Hence, the inventory level at time t, is governed by

$$\frac{dI_3(t)}{dt}+\alpha t I_3(t)=-D_0\mu,\quad t_1\le t\le t_2, \tag{3}$$

with the boundary conditions $I_3(t_2)=0$.
The solution to (1) is

$$I_1(t)=\frac{(\beta-1)D_0}{\alpha}\Big(1-e^{\frac{-\alpha t^2}{2}}\Big),\quad t\in[0,\mu]. \tag{4}$$

Similarly, the solution to (2) with $I_2(\mu)=I_1(\mu)=\frac{(\beta-1)D_0}{\alpha}(1-e^{\frac{-\alpha\mu^2}{2}})$ is

$$I_2(t)=e^{\frac{-\alpha t^2}{2}}\int_{\mu}^{t}(\beta-1)D_0\mu e^{\frac{\alpha w^2}{2}}dw+\frac{e^{\frac{-\alpha t^2}{2}}(\beta-1)D_0}{\alpha}\Big(e^{\frac{\alpha\mu^2}{2}}-1\Big),\quad t\in[\mu,t_1]. \tag{5}$$

Finally, the solution to (3) is

$$I_3(t) = e^{\frac{-\alpha t^2}{2}} \int_t^{t_2} D_0 \mu e^{\frac{\alpha}{2}w^2} dw, \quad t \in [t_1, t_2]. \tag{6}$$

Since $I_2(t_1) = I_3(t_1)$, we have

$$e^{\frac{-\alpha t_1^2}{2}} \int_\mu^{t_1} (\beta - 1) D_0 \mu e^{\frac{\alpha w^2}{2}} dw + \frac{e^{\frac{-\alpha t_1^2}{2}} (\beta - 1) D_0}{\alpha} \left(e^{\frac{\alpha \mu^2}{2}} - 1 \right) = e^{\frac{-\alpha t_1^2}{2}} \int_t^{t_2} D_0 \mu e^{\frac{\alpha}{2}w^2} dw,$$

$$\text{which implies } \beta\mu \int_\mu^{t_1} e^{\frac{\alpha w^2}{2}} dw + \frac{\beta - 1}{\alpha} \left(e^{\frac{\alpha \mu^2}{2}} - 1 \right) = \mu \int_\mu^{t_2} e^{\frac{\alpha}{2}w^2} dw. \tag{7}$$

From Eq.(7), it is obvious that t_2 is a function of t_1. As a result, the problem here has only one decision variable t_1. By taking the derivative of (7) with respect to t_1, we obtain

$$\frac{dt_2}{dt_1} = \beta e^{\frac{\alpha(t_1^2 - t_2^2)}{2}} > 0.$$

The total cost per cycle consists of the following three elements:

(a) The setup cost is A. (8)

(b) The production cost is

$$\begin{aligned} PC_1 &= \int_0^\mu \alpha_1 (D_0 t)^{-\gamma} \beta (D_0 t) dt + \int_\mu^{t_1} \alpha_1 (D_0 \mu)^{-\gamma} \beta (D_0 \mu) dt \\ &= \frac{\alpha_1 \beta D_0^{1-\gamma} \mu^{2-\gamma}}{2 - \gamma} + \alpha_1 \beta (D_0 \mu)^{1-\gamma} (t_1 - \mu). \end{aligned} \tag{9}$$

Here, owing the first term must be positive, which implies $\gamma < 2$.

(c) The inventory holding cost is

$$\begin{aligned} HC_1 &= \int_0^\mu c_1 I_1(t) dt + \int_\mu^{t_1} c_1 I_2(t) dt + \int_\mu^{t_2} c_1 I_3(t) dt \\ &= \int_0^\mu c_1 \frac{(\beta - 1) D_0}{\alpha} \left(1 - e^{\frac{-\alpha t^2}{2}} \right) dt \\ &\quad + \int_\mu^{t_1} c_1 \left[e^{\frac{-\alpha t^2}{2}} \int_\mu^t (\beta - 1) D_0 \mu e^{\frac{\alpha w^2}{2}} dw + \frac{e^{\frac{-\alpha t^2}{2}} (\beta - 1) D_0}{\alpha} \left(e^{\frac{\alpha \mu^2}{2}} - 1 \right) \right] dt \\ &\quad + \int_{t_1}^{t_2} c_1 \left[e^{\frac{-\alpha t^2}{2}} \int_t^{t_2} D_0 \mu e^{\frac{\alpha}{2}w^2} dw \right] dt. \end{aligned} \tag{10}$$

Therefore, the total cost per unit time during time-span $[0, t_2]$ is

$$TAC_1(t_1, t_2) = \frac{TC_1(t_1, t_2)}{t_2}, \tag{11}$$

where

$$
\begin{aligned}
TC_1(t_1,t_2) =& A + PC_1 + HC_1 \\
=& A + \frac{\alpha_1\beta D_0^{1-\gamma}\mu^{2-\gamma}}{2-\gamma} + \alpha_1\beta(D_0\mu)^{1-\gamma}(t_1-\mu) + \int_0^{\mu} c_1\frac{(\beta-1)D_0}{\alpha}\left(1-e^{\frac{-\alpha t^2}{2}}\right)dt \\
&+ \int_{\mu}^{t_1} c_1\left[e^{\frac{-\alpha t^2}{2}}\int_{\mu}^{t}(\beta-1)D_0\mu e^{\frac{\alpha w^2}{2}}dw + \frac{e^{\frac{-\alpha t^2}{2}}(\beta-1)D_0}{\alpha}\left(e^{\frac{\alpha\mu^2}{2}}-1\right)\right]dt \\
&+ \int_{t_1}^{t_2} c_1\left[e^{\frac{-\alpha t^2}{2}}\int_{t}^{t_2} D_0\mu e^{\frac{\alpha}{2}w^2}dw\right]dt. \qquad (12)
\end{aligned}
$$

Due to the fact that t_2 is a function of t_1, thus $TAC_1(t_1,t_2)$ in (11) can be reduced as a function of t_1, we denoted it by $TAC_1(t_1)$, i.e., $TAC_1(t_1) \equiv TAC_1(t_1,t_2)$. Hence, the problem faced by the vendor in Model 1 is

$$
\text{(P1)} \quad \text{Minimize } TAC_1(t_1) \qquad (13)
$$

$$
\text{subject to: } 0 < \mu < t_1, \qquad (13a)
$$

To minimize the total cost per unit time, taking the first derivative of $TAC_1(t_1)$ with respect to t_1, and setting the result to be zero, we obtain

$$
\begin{aligned}
\frac{dTAC_1(t_1)}{dt_1} &= \frac{1}{t_2}\left[\frac{dTC_1(t_1)}{dt_1} - TAC_1(t_1)\frac{dt_2}{dt_1}\right] \\
&= \frac{1}{t_2}\left[\frac{dTC_1(t_1)}{dt_1} - TAC_1(t_1)\beta e^{\frac{\alpha}{2}(t_1^2-t_2^2)}\right] = 0. \qquad (14)
\end{aligned}
$$

Let t_1^* denoted the optimal value of t_1, then t_1^* must satisfy Eq. (14). Furthermore, we can see that the stationary point t_1^* also satisfies the sufficient condition $\frac{d^2TAC_1(t_1)}{dt_1^2}|_{t=t_1^*} > 0$ from the given values of parameters in next section.

Consequently, we can obtain the value of t_1^* from Eq. (14) by using Newton's method or any commercial software, e.g., the subroutine *FindRoot* in Mathematica 5.0. Although we are unable to prove that the solution to Eq. (14) uniquely exits, the numerical examples in section 4 below indicate so. Once the optimal solution t_1^* is obtained, the corresponding optimal value t_2^* can be determined from Eq. (7).

Model 2. Model with partial backlogging

In this model, the behaviour of inventory system is depicted in Figure 3.

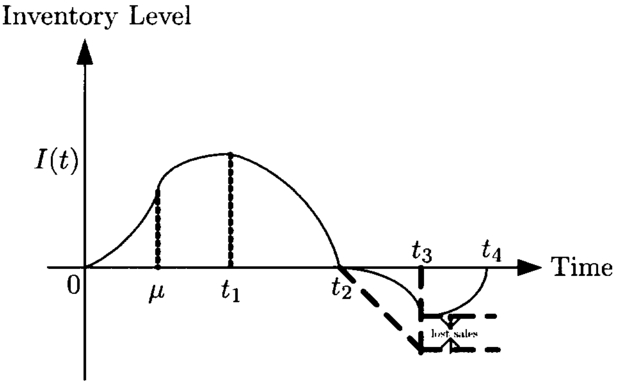

Figure 3. Model 2: The inventory model with shortage.

The Phases 1, 2 and 3 are same as those in Model 1.

Phase 4. At a given time $t \in [t_2, t_3]$ only a fraction of demand is backlogged. And the backlogging rate $b(\tau) = k_0 e^{-k_1 \tau}$, $k_0 \leq 1$, $0 \leq k_1$ is a function of the waiting time $\tau = t_3 - t$. Therefore, the backorders level at time $t \in [t_2, t_3]$, is governed by the following differential equation:

$$\frac{dI_4(t)}{dt} = -D_0 \mu b(\tau) = -D_0 \mu k_0 e^{-k_1(t_3-t)}, \quad t_2 \leq t \leq t_3, \tag{15}$$

with the boundary condition $I_4(t_2) = 0$.

Phase 5. For $t \in [t_3, t_4]$, the backorders level at time t, is governed by the following differential equation:

$$\frac{dI_5(t)}{dt} = (\beta - 1) D_0 \mu, \quad t_3 \leq t \leq t_4, \tag{16}$$

with the boundary condition $I_5(t_4) = 0$.

The solution to (15) is

$$I_4(t) = \int_t^{t_2} D_0 \mu k_0 e^{-k_1(t_3-w)} dw = \frac{D_0 \mu k_0}{k_1} \left[e^{-k_1(t_3-t_2)} - e^{-k_1(t_3-t)} \right], \quad t \in [t_2, t_3]. \tag{17}$$

Setting $t = t_3$ into Eq.(17), we obtain the maximum number of backorders per cycle as follows:

$$-I_4(t_3) = \frac{D_0 \mu k_0}{k_1} \left[1 - e^{-k_1(t_3-t_2)} \right]. \tag{18}$$

The solution to (16) is

$$I_5(t) = (\beta - 1) D_0 \mu (t - t_4), \quad t \in [t_3, t_4]. \tag{19}$$

Given the condition $I_4(t_3) = I_5(t_3)$, we get

$$\frac{D_0\mu k_0}{k_1}\Big[1 - e^{-k_1(t_3-t_2)}\Big] = (\beta - 1)D_0\mu(t_4 - t_3). \tag{20}$$

$$\text{which implies } t_4 = t_3 + \frac{k_0}{(\beta-1)k_1}\Big[1 - e^{-k_1(t_3-t_2)}\Big]. \tag{21}$$

The total cost per cycle of the system consists of the following five elements:

(a) The setup cost is A. (22)

(b) The production cost is

$$\begin{aligned} PC_2 &= \int_0^{\mu} \alpha_1(D_0t)^{-\gamma}\beta(D_0t)dt + \int_{\mu}^{t_1} \alpha_1(D_0t)^{-\gamma}\beta(D_0\mu)dt + \int_{t_3}^{t_4} \alpha_1(D_0\mu)^{-\gamma}\beta(D_0\mu)dt \\ &= \frac{\alpha_1\beta D_0^{1-\gamma}\mu^{2-\gamma}}{2-\gamma} + \alpha_1\beta(D_0\mu)^{1-\gamma}(t_1-\mu) + \alpha_1\beta(D_0\mu)^{1-\gamma}(t_4-t_3). \end{aligned} \tag{23}$$

(c) The inventory holding cost is

$$\begin{aligned} HC_2 &= HC_1 \\ &= \int_0^{\mu} c_1\frac{(\beta-1)D_0}{\alpha}\Big(1 - e^{\frac{-\alpha t^2}{2}}\Big)dt \\ &\quad + \int_{\mu}^{t_1} c_1\Big[e^{\frac{-\alpha t^2}{2}}\int_{\mu}^{t}(\beta-1)D_0\mu e^{\frac{\alpha w^2}{2}}dw + \frac{e^{\frac{-\alpha t^2}{2}}(\beta-1)D_0}{\alpha}\Big(e^{\frac{\alpha\mu^2}{2}} - 1\Big)\Big]dt \\ &\quad + \int_{t_1}^{t_2} c_1\Big[e^{\frac{-\alpha t^2}{2}}\int_{t}^{t_2} D_0\mu e^{\frac{\alpha}{2}w^2}dw\Big]dt. \end{aligned} \tag{24}$$

(d) The shortage cost for backlogged items is given by

$$\begin{aligned} BC_2 &= \int_{t_2}^{t_3} c_2[-I_4(t)]dt + \int_{t_3}^{t_4} c_2[-I_5(t)]dt \\ &= \int_{t_2}^{t_3} c_2\frac{D_0\mu k_0}{k_1}\Big[e^{-k_1(t_3-t)} - e^{-k_1(t_3-t_2)}\Big]dt + \int_{t_3}^{t_4} c_2(\beta-1)D_0\mu(t_4-t)dt \\ &= \frac{c_2D_0\mu k_0[1 - e^{-k_1(t_3-t_2)}(1 + k_1t_3 - k_1t_2)]}{k_1^2} + \frac{c_2(\beta-1)D_0\mu(t_4-t_3)^2}{2}. \end{aligned} \tag{25}$$

(e) The cost of lost sales is given by

$$\begin{aligned} LC_2 &= \int_{t_2}^{t_3} c_3[1 - b(\tau)]D_0\mu dt = \int_{t_2}^{t_3} c_3[1 - k_0e^{-k_1(t_3-t)}]D_0\mu dt \\ &= c_3D_0\mu\Big[(t_3-t_2) - \frac{k_0}{k_1}\Big(1 - e^{-k_1(t_3-t_2)}\Big)\Big]. \end{aligned} \tag{26}$$

Consequently, the total cost per unit time during time-span $[0, t_4]$ is

$$TAC_2(t_1, t_2, t_3, t_4) = \frac{TC_2(t_1, t_2, t_3, t_4)}{t_4}, \tag{27}$$

where

$$\begin{aligned}
&TC_2(t_1, t_2, t_3, t_4) \\
&= A + PC_2 + HC_2 + BC_2 + LC_2 \\
&= A + \frac{\alpha_1 \beta (D_0)^{1-\gamma} \mu^{2-\gamma}}{2-\gamma} + \alpha_1 \beta (D_0 \mu)^{1-\gamma} (t_1 - \mu + t_4 - t_3) \\
&\quad + \int_0^{\mu} \frac{c_1 (\beta - 1) D_0}{\alpha} \Big(1 - e^{\frac{-\alpha t^2}{2}}\Big) dt \\
&\quad + \int_{\mu}^{t_1} c_1 \Big[e^{\frac{-\alpha t^2}{2}} \int_{\mu}^{t} (\beta - 1) D_0 \mu e^{\frac{\alpha w^2}{2}} dw + \frac{e^{\frac{-\alpha t^2}{2}} (\beta - 1) D_0}{\alpha} \Big(e^{\frac{\alpha \mu^2}{2}} - 1\Big)\Big] dt \\
&\quad + \int_{t_1}^{t_2} c_1 \Big[e^{\frac{-\alpha t^2}{2}} \int_{t}^{t_2} D_0 \mu e^{\frac{\alpha}{2} w^2} dw\Big] dt \\
&\quad + \frac{c_2 D_0 \mu k_0 [1 - e^{-k_1 (t_3 - t_2)} (1 + k_1 t_3 - k_1 t_2)]}{k_1^2} + \frac{c_2 (\beta - 1) D_0 \mu (t_4 - t_3)^2}{2} \\
&\quad + c_3 D_0 \mu \Big[(t_3 - t_2) - \frac{k_0}{k_1} \Big(1 - e^{-k_1 (t_3 - t_2)}\Big)\Big].
\end{aligned} \tag{28}$$

From Eqs. (7) and (21), we know that t_2 is a function of t_1, and t_4 is a function of t_2 and t_3. Consequently, the decision variables in Model 2 can be reduced from four dimensions (t_1, t_2, t_3, t_4) to two dimensions (t_1, t_3), i.e., the problem faced by the vendor in this model is

$$\text{(P2)} \quad \text{Minimize } TAC_2(t_1, t_3) \tag{29}$$

$$\text{subject to: } 0 < \mu < t_1, \tag{29a}$$

$$t_1 < t_2 < t_3. \tag{29b}$$

Our objective is to find the optimal values of t_1 and t_3 such that $TAC_2(t_1, t_3)$ has minimum. The study procedure is similar to Model 1. That is, in order to find the optimal values of t_1 and t_3, we have to solve the complex nonlinear equations $\partial TAC_1(t_1, t_3)/\partial t_1 = 0$ and $\partial TAC_1(t_1, t_3)/\partial t_3 = 0$. Although it is difficult to solve the problem analytically, we can obtain the optimal values t_1^* and t_3^* by using Newton-Raphson Method (or any bisection method). Once we obtain the optimal value (t_1^*, t_3^*), the optimal solution (t_2^*, t_4^*) is obtained from Eqs.(7) and (21).

4. Numerical examples and sensitivity analysis

In this section, we adopt the partly values of data in Manna and Chaudhuri [9] for fitting the models. Meanwhile, we apply the Taylor's series expansion to above equations, by the assumption $\alpha << 1$ and neglecting the second and higher order of α terms. And we also use the subroutine *FindRoot* in Mathematica 5.0 to find the optimal values for our models respectively.

Example 1. For the model without shortage, we let $c_1 = 4$, $D_0 = 10000$, $\beta = 2.5$, $\alpha = 0.01$, $\alpha_1 = 15000$, $\gamma = 1.2$, $A = 100$ and $\mu = 12/360$ in appropriate units. By using the subroutine *FindRoot* in Mathematica 5.0, we obtain the optimal solutions for t_1, t_2 and TAC_1 are given as $t_1^* = 0.360279$, $t_2^* = 0.874777$ and $TAC_1^* = 5395.88$.

Example 2. For the model with shortage, we let $c_1 = 4$, $c_2 = 6$, $c_3 = 8$, $\beta = 4$, $D_0 = 10000$, $\alpha = 0.01$, $\alpha_1 = 15000$, $\gamma = 1.2$, $A = 100$, $k_0 = 0.9$, $k_1 = 0.8$ and $\mu = 0.12$ in appropriate units. By using the subroutine *FindRoot* in Mathematica 5.0, we obtain the optimal solutions for t_1, t_2, t_3, t_4 and TAC_2 are given as $t_1^* = 0.225503$, $t_2^* = 0.721449$, $t_3^* = 0.942117$, $t_4^* = 1.0028$ and $TAC_2^* = 6025.28$.

For studying the sensitivity analysis of the parameters on the proposed models, we changed (increasing or decreasing) the parameters by 25% and 50% and took one parameter at a time, kept the remaining parameters at their original values. The results are shown in Tables 1 and 2.

The Tables 1 and 2 reveal the following points.

(1) In Model 1, the optimal total cost per unit time, TAC_1^*, increases while c_1, α, β, α_1 and A increase. But this trend is reversed for parameters D_0, μ and γ. A simple economic interpretation is as follows: A larger value of c_1 indicates a larger unit holding cost per unit time, which leads to the optimal production run time (t_1^*) and the inventory cycle time (t_2^*) are decreasing, while the optimal total cost per unit time is getting larger; A larger value of β indicates a larger production rate which leads to many items remaining in stock. As a result, the inventory holding cost and the optimal total cost per unit time are getting larger; A larger value of γ indicates a smaller unit production cost, while the optimal total cost per unit time is getting smaller.

Table 1. Without shortage model (i.e., Model 1)

Changing parameter	(%) change	t_1^*	t_2^*	TAC_1^*	(%) change in TAC_1^*
c_1	+50	0.297839	0.719089	5547.29	+2.81
	+25	0.324543	0.785692	5475.43	+1.47
	−25	0.411527	1.002430	5305.66	−1.67
	−50	0.493691	1.206800	5199.30	−3.64
D_0	+50	0.290308	0.700298	5157.83	−4.41
	+25	0.320003	0.774369	5258.36	−2.55
	−25	0.418892	1.020760	5596.62	+3.72
	−50	0.514039	1.257350	5924.66	+9.80
μ	+50	0.338359	0.807681	5279.83	−2.15
	+25	0.347376	0.836388	5318.21	−1.44
	−25	0.379864	0.929797	5540.68	+2.68
	−50	0.412733	1.017870	5819.46	+7.85
α	+50	0.357966	0.868565	5398.42	+0.05
	+25	0.359113	0.871644	5397.15	+0.02
	−25	0.361465	0.877965	5394.59	−0.02
	−50	0.362672	0.881210	5393.30	−0.05
β	+50	0.267873	0.957348	5635.18	+4.43
	+25	0.303600	0.912212	5524.34	+2.38
	−25	0.471233	0.868215	5236.28	−2.96
	−50	0.884399	1.100550	4992.31	−7.48
α_1	+50	0.411678	1.002800	7860.10	+45.67
	+25	0.387181	0.941801	6629.84	+22.87
	−25	0.330400	0.800295	4157.12	−22.96
	−50	0.296666	0.716161	2911.85	−46.04
γ	+50	0.256317	0.615474	623.34	−88.45
	+25	0.284613	0.686089	1359.08	−74.81
	−25	0.444633	1.084820	27802.6	+415.26
	−50	0.268944	0.646991	153964.	+2753.37
A	+50	0.386647	0.940471	5450.96	+1.02
	+25	0.373712	0.908248	5423.92	+0.52
	−25	0.346288	0.839907	5366.72	−0.54
	−50	0.331666	0.803452	5336.29	−1.10

Table 2. With shortage model (i.e., Model 2)

Changing parameter	(%) change	t_1^*	t_2^*	t_3^*	t_4^*	TAC_2^*	(%) change in TAC_2^*
c_1	+50	0.205380	0.641124	0.728686	0.754056	6780.38	+12.53
	+25	0.222139	0.708023	0.784264	0.806453	6560.31	+8.88
	−25	0.276990	0.926758	0.974301	0.988297	5991.69	−0.56
	−50	0.330069	1.138040	1.166280	1.174660	5600.40	−7.05
c_2	+50	0.230424	0.741085	0.909788	0.957132	6096.68	+1.19
	+25	0.228246	0.732393	0.923689	0.976903	6065.07	+0.66
	−25	0.221955	0.707287	0.967512	1.037990	5973.80	−0.85
	−50	0.217208	0.688340	1.004200	1.087930	5904.94	−2.00
c_3	+50	0.235407	0.760964	0.906097	0.947204	6169.00	+2.39
	+25	0.231087	0.743731	0.921789	0.971576	6106.31	+1.34
	−25	0.218126	0.692007	0.970975	1.045980	5918.27	−1.78
	−50	0.208014	0.651641	1.018470	1.113840	5771.64	−4.21
k_0	+50	0.188277	0.572825	0.848389	0.959676	5485.65	−8.96
	+25	0.207342	0.648958	0.906643	0.993965	5761.90	−4.37
	−25	0.241328	0.784585	0.930362	0.961321	6254.96	+3.81
	−25	0.121398	0.305554	0.520405	0.550016	7515.48	+24.74
k_1	+50	0.228056	0.731634	0.928668	0.981310	6062.31	+0.61
	+25	0.226846	0.726808	0.935111	0.991523	6044.76	+0.32
	−25	0.224007	0.715477	0.949694	1.015250	6003.57	−0.36
	−50	0.222335	0.708804	0.957821	1.028930	5979.31	−0.76
D_0	+50	0.193346	0.593071	0.729614	0.768419	6235.89	+3.50
	+25	0.207224	0.648487	0.817427	0.864833	6131.43	+1.76
	−25	0.250400	0.820767	1.142550	1.227660	5916.85	−1.80
	−50	0.280181	0.939475	1.624980	1.783280	5775.96	−4.14
μ	+50	0.246318	0.714712	0.887933	0.936460	6734.40	+11.77
	+25	0.235822	0.717731	0.909774	0.963180	6377.70	+5.85
	−25	0.214511	0.722475	0.996011	1.069710	5688.47	−5.59
	−50	0.196895	0.697063	1.128640	1.238120	5384.45	−10.64
α	+50	0.225277	0.720264	0.941093	1.001820	6026.52	+0.02
	+25	0.225390	0.720855	0.941603	1.002310	6025.91	+0.01
	−25	0.225617	0.722045	0.942632	1.003298	6024.66	−0.01
	−50	0.225732	0.722645	0.943151	1.003796	6024.03	−0.02
β	+50	0.190149	0.839951	1.148020	1.19717	6770.45	+12.37
	+25	0.204706	0.782781	1.048270	1.102080	6422.79	+6.60
	−25	0.259134	0.657006	0.829368	0.901821	5551.32	−7.87
	−50	0.330772	0.601298	0.719168	0.719168	4936.92	−18.06
α_1	+50	0.252563	0.829393	1.170050	1.259500	8240.26	+36.76
	+25	0.240296	0.780467	1.057560	1.132120	7150.70	+18.68
	−25	0.207631	0.650110	0.818646	0.865946	4856.13	−19.16
	−50	0.185526	0.561835	0.679851	0.713635	3626.79	−39.81
γ	+50	0.132093	0.348302	0.37413	0.381799	1089.76	−81.91
	+25	0.158631	0.454379	0.517526	0.536000	1853.76	−69.23
	−25	–	–	–	–	–	–
	−50	–	–	–	–	–	–
A	+50	0.228903	0.735015	0.962100	1.024395	6074.61	+0.82
	+25	0.227212	0.728269	0.952156	1.013650	6050.07	+0.41
	−25	0.223776	0.714554	0.931979	0.991849	6000.21	−0.42
	−50	0.222028	0.707581	0.921740	0.980785	5974.86	−0.84

Note that: "–" denotes no optimal solution

(2) In Model 2, the optimal total cost per unit time, TAC_2^*, increases while c_1, c_2, c_3, α, β, α_1, k_1, D_0, μ and A increase. But this trend is reversed for parameters γ and k_0. A simple economic interpretation is as follows: A larger value of c_1 indicates a larger unit holding cost per unit time, which leads to the optimal production run time (t_1^*), the time at which shortages start to occur (t_2^*), the production restarting time (t_3^*) and the inventory cycle time (t_4^*) are decreasing, while the optimal total cost per unit time is getting larger; A larger value of c_2 indicates a larger unit shortage cost per unit time for backlogged items, which leads to t_1^* and t_2^* are increasing, t_3^* and t_4^* are decreasing, while the optimal total cost per unit time is getting larger; A larger value of c_3 indicates a larger unit cost of lost sales, which leads to t_1^* and t_2^* are increasing, t_3^* and t_4^* are decreasing, while the optimal total cost per unit time is getting larger; A lower of k_0 (or the larger value of k_1) indicates a lower backlogged rate, i.e., losing more customers to buy the items, while the optimal total cost per unit time is getting larger.

(3) It is seen that the percentage change in the optimal total cost per unit time is highly sensitive in parameters α_1 and γ, (especially in Model 1, low down to change 50% in γ, in which the unit production cost v will be 459.583 as the time t beyond the given time μ) and almost insensitive to changes in parameter α and moderately sensitive to changes in other parameters.

5. Concluding remarks

In this article, we extended Manna and Chaudhuri's [9] model by considering the backlogging rate, which is a negative exponential decreasing function of the waiting time until the next replenishment. Meanwhile, we also add the setup cost into the model, and reduce half of the unnecessary decision variables. Two numerical examples are given to illustrate the solution procedure and sensitivity analysis have been shown in Tables 1 and 2. A future research may be to consider probabilistic demand in the problem. We also can consider the sale price- or stock-dependent demand rate.

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