無模型設定隱含波動性模型預測績效—台指選擇權市場實證

Forecasting Performance of Model-free Implied Volatility for the Taiwan Option Market

許美滿 Mei-Maun Hseu
致理技術學院財金系　助理教授
Department of Finance, Chihlee Institute of Techonology

鍾惠民 Huimin Chung
交通大學財務金融所 教授
Graduate Institute of Finance, National Chiao Tung University

(Received: Feb. 11, 2009; First Revision: Apr 6, 2009; Accepted: May 13, 2009)

摘 要

本文目的旨在以 Britten-Jones 與 Neuberger (2000)所推導之無模型設定隱含波動性(model-free implied volatility, MF-IV) 模型，檢定台指選擇權之資訊內涵。藉由 MF-IV 模型與其他波動性模型如 Black-Scholes 隱含波動性 (BS-IV)、歷史波動性模型與 GARCH (1,1) 模型之預測績效比較，本文發現 MF-IV 模型完全包含歷史波動性模型與 GARCH (1,1) 模型在預測未來 5 天買權到期時現貨市場真實波動性所具有之資訊。

關鍵詞：無模型設定隱含波動性、台股期貨與台指選擇權

Abstract

This paper aims to examine an estimator of model-free implied volatility (MF-IV), as derived by Britten-Jones and Neuberger (2000), investigating its information content within the index options market of Taiwan. Based upon a comparison between the forecasting performance of the MF-IV model and that of other volatility forecasting models, such as Black-Scholes implied volatility (BS-IV), historical volatility and GARCH volatility, our empirical results demonstrate that the MF-IV model possesses higher information efficiency, subsuming all information contained within the historical and GARCH (1,1) volatility models in the forecasting of future realized volatility for a weekly forecasting horizon.

Keywords: Model-free Implied Volatility; TX; TXO.

JEL classification: G14; G15.

1. Introduction

The issue of implied volatility has attracted considerable attention along with a wealth of research over the past two decades. The extant literature apparently provides support for the contention that the Black-Scholes implied volatility (BS-IV) model is more efficient than other time-series models, such as the historical and GARCH (1,1) volatility models, in terms of measuring future realized volatility.[1]

However, since the assumptions of the BS-IV model do not completely hold in the real world, the actual forecasting performance of implied volatility is likely to be unsatisfactory in cases where the model is misspecified. Britten-Jones and Neuberger (2000) therefore proposed an alternative implied volatility measure, which they refer to as model-free implied volatility (MF-IV); as opposed to being derived from any specific model, MF-IV is derived entirely from a no-arbitrage condition. Jiang and Tian (2005) further found that the MF-IV model was still valid even where jumps were evident in the price of the underlying asset. For emerging derivative market such as Taiwan equity market, the effects of market friction may cause the BS-IV model to be mis-pecified. This paper aims to examine whether the MF-IV model provides better

---

[1] From a review of 93 studies over the past two decades in which the forecasting of volatility was based upon various volatility measures, Poon and Granger (2003) found that the implied volatility model was superior to the historical volatility model with regard to the forecasting of realized volatility. Using data from 35 futures options markets on eight separate exchanges, Szakmary et al. (2003) also found that, whilst not being a completely unbiased predictor of future volatility, implied volatility again outperformed historical volatility as a predictor of subsequent realized volatility in the underlying futures prices over the remaining life of the option. Other study found that implied volatilities outperformed GARCH (1,1) in the forecasting of future realized volatility (Chang and Tabak, 2007).

information content in the index options market of Taiwan.

Since the analysis of the forecasting ability of volatility is reliant upon an accurate measure of realized volatility, it is becoming increasingly evident that a realized volatility estimator computed from high-frequency data, such as five-minute data, provides enormous improvements to the quality of measurement of actual volatility and forecast evaluation. In addition, weekly (H1), bi-weekly (H2), tri-weekly (H3) and monthly (H4) forecast horizons are considered major horizons for options pricing and portfolio management. We therefore use the sum of the squared five-minute returns on the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) to calculate realized volatility, and focus on these four major forecasting horizons to examine whether forecasting accuracy is affected by horizon length over the remaining life of the Taiwan Stock Exchange Capitalization Weighted Stock Index options (TXO) contracts.

We first compare the forecasting performance of the four volatility models based upon forecast errors, and then examine their relative information content using univariate and encompassing regressions based upon the four major horizons for options pricing and portfolio management.[2] Our empirical results, based upon high-frequency (five-minute) returns data for the calculation of realized volatility,

[2] The encompassing regression is applied to determine whether the information content of the historical volatility or GARCH (1,1) volatility models is subsumed by the BS-IV and/or MF-IV models.

provide a number of interesting findings; for example, we find that the implied volatility models apparently outperform the time-series models, with the MF-IV model in particular demonstrating greater informational efficiency and subsuming all information contained within the historical and GARCH (1,1) volatility models over the shortest forecasting horizon, as compared to the BS-IV model. The findings provide investors with alternative IV measure for making their investment strategy in Taiwan options market.

The remainder of this paper is organized as follows. Section 2 provides a description of the institutional setting and data used this study, followed in Section 3 by an explanation of the methodology adopted. Section 4 presents and explains the empirical results. Finally, the conclusions drawn from this study are presented in Section 5.

2. Institutional Setting and Data

A TXO contract, a European-style option introduced by the Taiwan Futures Exchange (TAIFEX) on 24 December 2001, has a monthly expiration cycle (similar to TAIEX futures, which are traded under the ticker symbol, TX) with the expiration day being the first business day after the third Wednesday (the last trading day) of the contract month. However, to reduce overnight risks and to increase the capital efficiency, the

Financial Supervisory Commission (FSC) on 11 September 2008 approved Taiwan Futures Exchange to modify the final settlement price methodology of stock index futures and options. The new methodology has been applied to December contracts of 2008, based on the arithmetic average price of the underlying index disclosed within the last 30 minutes prior to the close of trading on the last trading day. That is, the final settlement day is the same day as the last trading day.[3]

For the TXO contract, daily trading comprises of the spot month and the next two calendar months followed by two additional months from the March quarterly cycle (March, June, September and December).[4] Any option which is 'in-the-money' and which has not been liquidated or exercised on the last trading day is automatically exercised.

The TXO market has grown rapidly since its launch in 2001. Of the 168 exchange-traded index options worldwide, the TXO trading volume in 2005 ranked it as the third highest global trading volume, reaching a record 80,096,506 contracts in that year.[5] Since the trading volume of call options is larger than that of put options, this study compares the forecasting performance of the BS-IV, MF-IV, historical and GARCH (1,1) volatility models using the data on nearby TXO call contracts covering

3 In our studying period, the previous settlement methodology is applied.

4 Daily trading of TX contract comprises of the spot month and the next calendar month followed by three additional months from the March quarterly cycle.

5 As reported by the Futures and Options World (FOW) 2005 statistics.

the period from 24 December 2001 to 22 December 2005. We select nearby option contracts because they are the most actively traded option contracts within their own classification, which therefore minimizes the problem of infrequent trading.

Our study sample comprises of 191 observations from the various volatility models for the prediction of future realized volatility for the weekly (H1), bi-weekly (H2), tri-weekly (H3) and monthly (H4) forecasting horizons.[6] We use high-frequency (five-minute) natural log return data from the TAIEX to calculate realized volatility, and use the daily natural log return of the TAIEX to calculate both historical volatility and GARCH (1,1) volatility.

As regards implied volatility, we calculate BS-IV by considering the practical investment phenomenon of TXO investors, whose investment decisions are invariably based upon the TX market situation, and then calculate the implied spot prices by using the closing prices of the corresponding TX contracts and using these as proxies for the spot indices of the TXO which are closest to 'at-the-money' for nearby contracts.[7] As for the MF-IV calculation, because 'in-the-money' options are more expensive and less liquid than 'at-the-money' or 'out-of-the-money' options, following

[6] Although there should be a total of 192 observations under the (H1), (H2), (H3) and (H4) forecast horizons for the full 48 expiration months covered in this study, there is no H4 horizon since there were only 14 trading days during this period between the expiration months of January and February 2005.

[7] Using implied spot prices to calculate implied volatility will be justified only when the futures contracts and options contracts expire on the same day. Both our near TX and TXO contracts satisfy this condition.

Jiang and Tian (2005), we exclude from our sample all call options with strike prices of less than 97 per cent of the implied spot prices of the underlying asset.

The trading data on the TXO, TX and TAIEX matching the abovementioned volatility calculation are obtained from the Taiwan Economic Journal (TEJ) databank. The data on the TXO and TX covers the period from 24 December 2001 to 22 December 2005, whilst that for the TAIEX runs from 1 September 1998 to 22 December 2005. We use the fixed rate of the one-year time deposits offered by First Commercial Bank as the proxy for the risk-free rate.

3. Methodology

Given that realized volatility is not directly observable, it must be estimated. It is argued in numerous studies that a realized volatility estimator which is computed from high-frequency data, such as five-minute data, provides enormous improvements to the measurement quality for actual volatility yield and forecast evaluation.[8] Bandi and Russell (2003) also argue that in the presence of market microstructure noise, five-minute sampling frequency is close to the optimal. Thus, we use the sum of the squared five-minute returns on the TAIEX to calculate realized volatility.[9] Assuming that time is measured in trading days, and that there are 252 trading days per year,

8 See for example, Anderson and Bollerslev (1998), Andersen (2000), Andersen et al. (2001), Andreou and Ghysels (2002) and Barndorff-Nielsen and Shephard (2001, 2002)

9 For example, in our paper, returns are sampled every 5-minute between the trading hours of 9:00 a.m. and 1:30 p.m. corresponding to the 54 intervals of the TAIEX within a single trading day.

realized volatility per annum can be calculated as:

$$\sigma_t^{RV} = \sqrt{\sum_{i=1}^{54} r_{it}^2} \times \sqrt{252} \qquad (1)$$

where r_{it} is the TAIEX five-minute intra-day natural log return at interval i of day t.

As noted by Ghysels et al. (2006), weekly, bi-weekly, tri-weekly and monthly forecasting horizons are the major horizons for options pricing and portfolio management. We therefore focus on the predicting ability of future realized volatility based upon these four TXO nearest to expiration days. The four volatility estimators in this study are tested against realized volatility over the remaining life of TXO contracts by means of forecast error and regression analysis. These estimators are calculated from time-series models (the historical and GARCH (1,1) volatility models) and implied volatility models (the BS-IV model and MF-IV models). The former are econometrics model which are based on historical data, whereas the latter are based on options market prices.

3.1 Historical Volatility

Historical volatility, which is perhaps the oldest and simplest of all volatility models, parameterizes current volatility as:

$$\sigma_t^{HV} = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N} r_t^2} \times \sqrt{252} \qquad (2)$$

where r_{it} is the natural log of the ratio of the TAIEX from the current day (t) to the previous day ($t-1$). Any observations inside the window of size N are given an equal weighting of $1/(N-1)$; in other words, volatility is forecasted as being identical to the last N periods. As noted by Kroner (1996), if the dataset used to construct this estimate is too large, then there is an inherent risk of the estimation being clouded by stale data. On the other hand, if insufficient observations are used to construct the estimate, then there is the alternative risk of the volatility estimates being dominated by one or two observations. As noted by ap Gwilym (2001), a simple 20-day historical estimator has been found to perform well for short forecast horizons; therefore, in this study, we use the last 20-day data to calculate historical volatility.

3.2 The GARCH(1,1) Model

Financial time-series returns often exhibit characteristics of time-varying volatility levels and volatility clustering which cannot be captured by the historical volatility model. Engle (1982) proposed an ARCH model which allows conditional variances to change over time; however, it was found that a practical problem in fitting ARCH (p) models to financial returns data was that the order 'p' needed to be fairly large if a good fitting model was to be obtained. Bollerslev (1986) extended the ARCH model to the GARCH model, which provides more parsimonious results than those of the ARCH model, and which has since become widely used for effectively dealing with volatility clustering and fat tailing phenomena in the equity returns, particularly with regard to the GARCH (1,1) model. This model can be defined as:

$$r_t = \mu_t + \varepsilon_t,\ \ \varepsilon_t = \eta_t \sigma_t,\ \ \sigma^{2\,GARCH}_{t} = \omega + \alpha\varepsilon^2_{t-1} + \beta\sigma^{2\,GARCH}_{t-1} \tag{3}$$

where $\omega > 0$, $\alpha \geq 0, \beta \geq 0$ are sufficient for $\sigma^{2\,GARCH}_{t} > 0$, and η_t are independently identically distributed (i.i.d.) random variables with zero mean and unit variance. The GARCH (1,1) model is estimated in this study using a rolling window of 866 daily TAIEX returns.

3.3 Black-Scholes Implied Volatility

The Black-Scholes (1973) option pricing model (B-S model) provides the foundation for the modern theory of options valuation; however, one variable which cannot be directly observed in this model is stock price volatility. If the option markets are efficient, Black-Scholes implied volatility (BS-IV) at time t (σ_t^{BS}) is inverted using the following BS-IV model:

$$\sigma_t^{BS} = f^{-1}(S_t, K, r, \tau, C_t^{MKT}) \tag{4}$$

where S_t is the underlying asset price at time t; K is the strike price; r is the risk-free interest rate; τ is the remaining time to maturity; and C_t^{MKT} denotes the market price of the option at time t.

Lee and Nayar (1993) note that "market makers in SPX options are continually hedging their positions with the companion S&P 500 futures contracts"; Draper and Fung (2002) also argue that by pricing the options contracts directly with the futures contracts, arbitrageurs could avoid high transaction and market-impact costs, as well as the inclusion of stale prices in the index arising from the non-trading of constituent stocks. Therefore, since TXO investors generally make their investment decisions based upon TX prices, as opposed to the TAIEX, we use the implied spot price as the proxy for S_t, which is inferred using the closing prices of the nearby TX contracts discounted at the risk-free rate, and use the closing prices of nearby TXO contracts which are closest to 'at-the-money' as the proxy for C_t^{MKT}. If markets are efficient, and the option pricing model is correct, then the implied volatility calculated from option prices should be an unbiased estimator of future realized volatility with informational efficiency; that is, it should correctly take in all of the available information, including the asset's price history.

3.4 Model-free Implied Volatility

It is well known that the test for the forecasting quality of implied volatility is in fact a joint test of the efficiency of the option markets and a specification of the options pricing model; therefore, the forecasting performance of the BS-IV model would be unsatisfactory if the model was misspecified. Britten-Jones and Neuberger (2000) propose an alternative implied volatility measure which, as opposed to being reliant upon a specific model, is derived entirely from no-arbitrage conditions. Since it does not impose strong distributional assumptions, the forecast is common to all consistent processes; hence, this model is regarded as model-free implied volatility (MF-IV).

Suppose that call options with a continuum of strike prices (K) for a given maturity (T) are traded on an underlying asset. Let the forward asset price be denoted as F_t, and the forward option price be denoted as $C^F(T,K)$. Following Dumas et al. (1998) and Britten-Jones and Neuberger (2000), Jiang and Tian (2005) provide a simpler derivation for MF-IV under the assumption of diffusion. The integrated return variance between current date, 0, and a future date, T, is fully specified by the set of call option prices expiring on date T. The MF-IV of Britten-Jones and Neuberger is thus defined as an integral of option prices over an infinite range of strike prices:

$$E_0^F\left[\int\left(\frac{dF_t}{F_t}\right)^2\right]=2\int_0^{\infty}\frac{C^F(T,K)-\max(0,F_0-K)}{K^2}dK \tag{5}$$

where the superscript F is the forward probability measure.

It is a straightforward matter to apply this model to stock prices under the assumption that the interest rate and dividends are deterministic. For the case of options on individual stocks or an individual index, let $C(T,K)$ denote the option price and S_t denote the underlying asset price at time t. We have $F_t = S_t/B(t,T)$ and $C^F(T,K) = C(T,K)/B(t,T)$, where $B(t,T)$ is the time t price of a zero coupon bond which pays \$1 at time T. Hence, MV-IV can be estimated using the following equation:

$$E_0^F\left[\int\left(\frac{dS_t}{S_t}\right)^2\right]=2\int_0^{\infty}\frac{C(T,K)/B(0,T)-\max(0,S_0/B(0,T)-K)}{K^2}dK \tag{6}$$

Since option exchanges offer only limit numbers of strike prices, the numerical integration of MV-IV can be implemented through the trapezoidal rule:

$$2\int_{K_{\min}}^{K_{\max}} \frac{C^*(T,K) - \max(0, S_0^* - K)}{K^2} dK = \sum_{i=1}^{m} [h(T,K_i) + h(T,K_{i-1})]\Delta K \qquad (7)$$

where $C^*(T,K) = C(T,K)/B(0,T)$; $S_0^* = S_0/B(0,T)$; $\Delta K = (K_{\max} - K_{\min})/m$, $K_i = K_{\min} + i\Delta K$ for $i = 0,\ldots,m$, and $h(T,K_i) = [C^*(T,K_i) - \max(0, S_0^* - K_i)]/K_i^2$.

In general, the MF-IV measure has several advantages over that of BS-IV. First, since there is no reliance upon any specific assumptions on the underlying asset price, the MF-IV model may avoid the estimation bias resulting from misspecifications, such as those associated with the BS-IV model. Second, by subsuming more information as a result of the consideration of more contracts, not only those that are 'at-the-money' as in the BS-IV model, MF-IV may have better forecasting performance than BS-IV. There are, however, problems with the MF-IV measure in emerging options markets, such as the numerical errors resulting from limited strike prices, the potential for MF-IV violating the boundary conditions of the options if there are numerous distortions in option prices as a result of specific demand, and the occasional existence of zero trading volume at certain strike prices. These 'discretization' errors may result in the unavailability of implied volatility, at which time the MF-IV measure will become biased; therefore, in an attempt to improve pricing efficiency, referring to Jiang and Tian (2005), we use a cubic spline in the curve-fitting of implied volatility, as opposed to option prices.

The prices of the listed calls are first translated into implied volatility based upon the Black-Scholes model, with a smooth function then being fitted to this implied volatility. We extract the implied volatility at strike prices K_i from the fitted function, with the Black-Scholes model again being used to invert the extracted implied volatility into call prices. Since these call prices exclude from our sample all call options with a strike price of less than 97 per cent of the implied spot price, MF-IV is calculated by using the RHS of Equation (7).

3.5 Volatility Forecast Evaluation Criteria

The three dominant methods for testing the competing estimates of future volatility are 'root mean squared error' (RMSE), 'mean absolute error' (MAE) and regressions. Fair and Shiller (1990) argue that in comparing alternative forecasts, RMSE is dominated by regression analyses; therefore, referring to the extant research, we employ the univariate analysis in Equation (8) and the encompassing regressions in Equations (9) and (10) to analyze the information content of the BS-IV and MF-IV forecast measures:

$$\sigma_t^{RV} = \alpha + \beta\sigma_t^{FV} + u_t \tag{8}$$

$$\sigma_t^{RV} = \alpha + \beta_1\sigma_t^{BS} + \beta_2\sigma_t^{FV1} + u_t \tag{9}$$

$$\sigma_t^{RV} = \alpha + \beta_1 \sigma_t^{MF} + \beta_2 \sigma_t^{FV1} + u_t \quad (10)$$

where σ_t^{RV} is the realized volatility at time t; σ_t^{FV} refers to the BS-IV, MF-IV historical and GARCH (1,1) volatility estimators, and σ_t^{FV1} expresses historical volatility and GARCH (1,1) volatility.

In a univariate regression, realized volatility is regressed on a single volatility forecast, examining the forecasting ability and information content of a single volatility forecast. Conversely, from an encompassing regression we can examine the relative importance of competing volatility forecast models, between BS-IV and historical volatility, between BS-IV and GARCH (1,1) volatility, between MF-IV and historical volatility, and between MS-IV and GARCH (1,1) volatility.[10] If the BS-IV or MF-IV models contain more information than the other volatility measures, we would expect a null hypothesis of H_0: $\beta_2 = 0$. If the joint hypothesis is H_0: $\beta_1 = 1$ and $\beta_2 = 0$, this indicates that the BS-IV or MF-IV measure has fully subsumed the information contained within the other volatility measures.

As noted in the prior studies, the volatility in the above equations contains measurement errors resulting from heteroskedasticity and serial correlation. Newey and West (1987) propose a general covariance estimator which remains consistent, even in the presence of both heteroskedasticity and autocorrelation of unknown form.

10 In order to avoid the problem of collinearity, no direct comparison is made between the relative performance of BS-IV and MF-IV by including them as regressors in the same regression.

Therefore, we use the Newey and West (1987) variance-covariance estimator in this study to correct for heteroskedasticity and serial correlation.

4. Empirical Results

4.1 Analysis of the Summary Statistics

Table 1 provides the summary statistics for the annualized volatility levels under various forecasting horizons covering the period from 24 December 2001 to 22 December 2005. As the table shows, the means of all these four measures are higher than the mean for realized volatility. Although historical volatility and realized volatility have roughly equal means, their standard deviations are, however, rather diverse. Since BS-IV has the highest means for the various forecast horizons, the argument of Jorion (1995), Fleming (1998) and Bates (2000), that the BS-IV measure provides upwardly biased forecasts, does seem to gain support from our results.

<Table 1 is inserted about here>

It is also difficult to judge from the maximum and minimum of BS-IV, MF-IV, historical volatility and GARCH (1,1) volatility, which is nearest to realized volatility; it is, however, worth noting that all of the maximum volatility estimators for these four measures occurred on the same day, 20 May 2004, which is the date of the inauguration of the 11^{th}-Term President and Vice President of Taiwan, together with the expiration day of the Taiwan index derivatives contracts, thereby increasing the expiration day effect in terms of return volatility levels.

4.2 Forecast Error Analysis

The results of MAE and RMSE are reported in Table 2, where the numbers in parentheses are the ranking value; a smaller ranking value indicates better forecasting ability for the model.

<Table 2 is inserted about here>

As indicated by Table 2, MF-IV performs the best, with the GARCH (1,1) measure ranking in second place, and most of the MAE and RMSE values for BS-IV and historical volatility producing the same ranking. This is consistent with our hypothesis that the MF-IV model may provide better forecasting performance than the BS-IV model in emerging derivative markets, such as the index options market of Taiwan, since the effects of market friction could well lead to some misspecification of the BS-IV model.[11] In terms of time series, the results showing that the GARCH (1,1) model outperforms the historical volatility model indicate the existence of volatility clustering and fat tailing in the Taiwanese equity market.

As reported in Table 1, in the measures provided by each of the BS-IV, MF-IV, historical and GARCH (1,1) volatility models, the maximum volatility estimations

[11] Examples of market friction in the Taiwanese stock market include the price limit rule, short-sale restrictions, transaction costs and index tracking errors.

occurred on 20 May 2004; however, we also find that the occurrence of maximum MAE on 20 May 2004 was only discernible between realized volatility and BS-IV, which is to be found in the (H4) monthly forecast horizon (Table 2). Our results seem to indicate that it is in fact BS-IV which may be biased, due to the presence of jumps, as opposed to MF-IV. Thus, the argument of Jiang and Tian (2005), that the MF-IV model remains valid even in those cases where the underlying asset prices have jumps, does seem to gain support from our results.

Appendix A reports the 2002-2005 observations on realized volatility and the forecast volatility levels for the various models for the (H4) monthly forecast horizon; it is worth noting that after 2004, the ratio of absolute error between MF-IV and BS-IV (RAEIV) reveals that the MF-IV measure appears to have lower forecasting errors. In order to check the robustness of our analysis, we further regress this RAEIV on the spread:

$$RAEIV_t = \alpha + \beta Spread_t + \varepsilon_t \tag{11}$$

where $RAEIV_t$ is the ratio of absolute error between MF-IV and BS-IV at time t and $Spread_t$ refers to the bid-ask spreads of the nearby TXO call contracts where the last buying and selling prices were both greater than zero.

Liquidity is improved as a direct result of the smaller spread, and, as a result of this improved liquidity, the performance of the MF-IV model will also be enhanced;

we therefore expect the sign of β to be positive. Table 3 shows that the coefficient of the bid-ask spread ($\hat{\beta}$) is insignificantly different from zero during period 1; it is, however, significantly positive at the 1 per cent level during period 2. Furthermore, the median of the bid-ask spreads (*Spread*) was 25.4094 during period 1, and 13.6654 during period 2.

The Wilcoxson rank-sum test also supports our earlier observation that, after 2004, there was a significant reduction in bid-ask spread (at the 1 per cent significance level). Thus, the MF-IV model appears to perform better with improvements in liquidity levels within the options market.

<Table 3 is inserted about here>

4.3 Univariate Regression Analysis

Table 4 reports the univariate regression results, showing that the coefficients of the various volatility measures are all significantly different from zero at the 1 per cent level, and that the Wald test statistics (χ^2-statictics) of the BS-IV, historical and GARCH(1,1) volatility models are highly significant for each of the various forecast horizons, thereby indicating rejection of the joint null hypothesis of $\alpha = 0$ and $\beta = 1$ in Equation (8). This implies that although the BS-IV, historical and GARCH (1,1) volatility models contain information on the forecasting of realized volatility, the estimations from such forecasting are biased.

<Table 4 is inserted about here>

On the other hand, the χ^2-statictics of the MF-IV model are insignificant, with the one exception of the (H2) bi-weekly forecast horizon, indicating that we cannot reject the joint null hypothesis of $\alpha = 0$ and $\beta = 1$. This implies that, as compared to the BS-IV, historical and GARCH (1,1) volatility models, and with the one exception of the (H2) bi-weekly forecast horizon, the MF-IV model may be regarded as an unbiased estimator for the forecasting of realized volatility.

The R^2-statictics show that, with the exception of the (H4) monthly forecast horizon, the BS-IV model has greater explanatory power than that of each of the other models, with historical volatility demonstrating the lowest explanatory power. Thus, as the results in Table 4 indicate, although the BS-IV model is biased, a strong relationship does, nevertheless, exist between this model and realized volatility.

4.4 Encompassing Regression Analysis

The results of the univariate regression show that, relative to the various time-series models, implied volatility models perform very well. We therefore go on to conduct an encompassing regression analysis, beginning with an exploration of the informational efficiency of the BS-IV model relative to the efficiency of the historical and GARCH (1,1) volatility models, through an examination of their respective encompassing regressions (Table 5). We then go on to examine the informational efficiency of the

MF-IV model relative to the historical and GARCH (1,1) volatility models.[12]

Table 5 reports the forecasting ability and information content of the BS-IV model. If this model provides greater information as compared to the historical and GARCH (1,1) volatility models, then we would expect to find the null hypothesis of H_0: $\beta_2^{HV} = 0$ in Panel A; and H_0: $\beta_2^{GARCH} = 0$ in Panel B. As Table 5 shows, it is only in the (H4) monthly forecast horizon that the historical and GARCH (1,1) volatility models contain more information.

For those periods which are shorter than the (H4) monthly forecast horizons, the information provided by the historical and GARCH (1,1) volatility models is already contained within the BS-IV model. In other words, the historical and GARCH (1,1) volatility models become redundant when each of these is regarded as a regressor for inclusion, together with the BS-IV measure, within the same regression. Furthermore, we also find an increase in explanatory power ($\overline{R}^2$-statictics) over the forecasting horizons. If the BS-IV model does have informational efficiency, subsuming all information contained in the alternative volatility forecast models, then we would expect the joint null hypothesis of H_0: $\beta_1^{BS} = 1$ and H_0: $\beta_2^{FV1} = 0$, where FV1= HV or GARCH (1,1) holds in all specifications.

<Table 5 is inserted about here>

---

12 To avoid the problem of multi-collinearity, we do not directly compare the relative efficiency between BS-IV and MF-IV by using the encompassing regression method.

As Table 5 shows, the Wald test statistics (χ^2-statictics) are significant for the various forecasting horizons in all of the encompassing regressions where the BS-IV coefficients are significantly different from zero, indicating that the joint null hypothesis of $\beta_1^{BS} = 1$ and $\beta_2^{HV} = 0$ (Panel A) or $\beta_1^{BS} = 1$ and $\beta_2^{GARCH} = 0$ (Panel B) does not hold. Our results imply that the BS-IV model has informational efficiency and subsumes part, but not all, of the information contained within the historical and GARCH (1,1) volatility forecasts.

Table 6 presents the results of the encompassing regression in those cases where the historical volatility measure (Panel A) or the GARCH (1,1) volatility measure (Panel B) is regarded as a regressor for inclusion, together with MF-IV, within the same regression. If the MF-IV model performs more efficiently than the historical or GARCH (1,1) volatility models in forecasting realized volatility, then we would expect to see all of the MF-IV coefficients (but not the historical or GARCH (1,1) volatility coefficients) being significantly different from zero in the respective encompassing regressions.

<Table 6 is inserted about here>

The results reveal that the only strong rejection of the null hypotheses of $\beta_2^{HV} = 0$ in Panel A; and $\beta_2^{GARCH} = 0$ in Panel B, arises in the encompassing regression on the (H4) monthly forecasting horizons. It is worth noting that for the (H1) weekly forecasting horizon, the joint null hypotheses hold for both $\beta_1^{MF} = 1$ and $\beta_2^{HV} = 0$ in Panel A, and $\beta_1^{MF} = 1$ and $\beta_2^{GARCH} = 0$ in Panel B. The results provide further evidence of the informational efficiency of the MF-IV model over shorter forecasting horizons, where it subsumes all of the information contained in the historical volatility and GARCH (1,1) volatility forecasts.

In order to examine whether the informational content of the implied volatility models may be biased due to jumps, all data for 20 May 2004 is excluded from the (H4) monthly forecasting horizon sample; this is then referred to as (H4A). As we can see from H4A in Table 5, although the BS-IV coefficient is still insignificant when the historical volatility measure is regarded as a regressor for inclusion together with BS-IV in Panel A, the BS-IV coefficient becomes significant at the 5 per cent level when the GARCH (1,1) volatility measure is regarded as a regressor for inclusion together with BS-IV in Panel B. Conversely, as we can see from H4A in Table 6, the MF-IV coefficient is still insignificant when either the historical or GARCH (1,1) volatility measures are regarded as regressors for inclusion together with the MF-IV measure.

Our results appear to provide support for Jiang and Tian (2005), that the MF-IV model remains valid even where there are jumps in the underlying asset prices. In general, the results of the univariate and encompassing regressions indicate that the implied volatility models outperform the time-series models. The BS-IV model demonstrates informational efficiency and subsumes most, but not all, of the information contained within the historical or GARCH (1,1) volatility models. However, whilst also demonstrating informational efficiency, the MF-IV model subsumes all of the information contained in the historical or GARCH (1,1) volatility models for the (H1) weekly forecasting horizon, thereby implying that the MF-IV model outperforms the BS-IV model over the shortest forecasting horizon for the remaining life of TXO contracts.

5. Conclusions

This paper compares the relative forecasting performance of BS-IV, MF-IV, historical volatility and GARCH (1,1) volatility estimators over four major forecasting horizons using data on nearby TXO call option contracts covering the period from 24 December 2001 to 22 December 2005. We investigate whether the MF-IV model provides better information content than the BS-IV model in an emerging market.

Following Jiang and Tian (2005), the MF-IV measure is calculated from observed option prices by employing a curve-fitting method based on a cubic smoothing spline and interpolation from endpoint implied volatility levels between the available strike prices. As noted by Jiang and Tian (2005), the MF-IV model considers the aggregate information level across options with different strike prices, whilst the analysis of the forecasting performance of the BS-IV model generally involves a joint test of market efficiency and the assumed specific option pricing model. Therefore, since no specific price dynamic is required, the MF-IV may well provide better information content.

Our results provide evidence to show that implied volatility models are more efficient than time-series models in the forecasting of realized volatility. The RMSE and MAE results show that the MF-IV model remains consistent with our hypotheses. The univariate regression results show that, as compared to the BS-IV, historical volatility and GARCH (1,1) volatility measures, the MF-IV measure may be regarded as an unbiased estimator for the forecasting of realized volatility. The encompassing regression analyses also suggest that the MF-IV model has informational efficiency, and that over the remaining life of TXO contracts on a weekly forecasting horizon, the model subsumes all of the information contained in the historical and GARCH (1,1) volatility estimators. This is consistent with ap Gwilym (2001), that the forecasting accuracy of all volatility models is affected by horizon length.

Although we also find that the BS-IV model contains richer information than any of the other volatility measures, it is, nevertheless, a biased estimator and subsumes only part of the information contained in the other measures. Our results also show that whilst the MF-IV measure remains unbiased in the presence of jumps, this is not the case for the BS-IV measure. Our findings provide support for the argument of Jiang and Tian (2005), that the MF-IV model remains valid even where there are jumps in the underlying asset price.

Since the effects of market friction in Taiwan, such as the price limit rule, short-sale restrictions, transaction costs and index tracking errors may lead to misspecification of the BS-IV model, our results are particularly informative for options investors in emerging derivative markets. Furthermore, the superiority of the MF-IV model over the BS-IV model, in terms of forecasting performance, is also enhanced by the improving liquidity levels in the TAIFEX market.

References

Andersen, T. G., and T. Bollerslev (1998), "Answering the skeptics: Yes, standard volatility models do provide accurate forecasts," *International Economic Review* 39, 885-905.

Andersen, T. G. (2000), "Some reflections on analysis of high-frequency data," *Journal of Business & Economic Statistics* 18, 146-153.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens (2001), "The distribution of realized stock return volatility," *Journal of Financial Economics* 61, 43-76.

Andreou, E., and E. Ghysels (2002), "Rolling-sample volatility estimators: Some new theoretical, simulation, and empirical results," *Journal of Business & Economic Statistics* 20, 363-376.

ap Gwilym, O. (2001), "Forecasting volatility for options pricing for the U.K stock market," *Journal of Financial Management and Analysis* 14, 55-62.

Bandi, F. M., and J. R. Russell (2003), "Microstructure noise, realized volatility, and optimal sampling," Working paper, University of Chicago Graduate School of Business.

Barndorff-Nielsen, O. E., and N. Shephard (2001), "Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics," *Journal of the Royal Statistical Society B* 63, 167-241.

Barndorff-Nielsen, O. E., and N. Shephard (2002), "Econometric analysis of realized volatility and its use in estimating stochastic volatility models," *Journal of Statistic Society* 64, 253-280.

Bates, D. S. (2000), "Post-'87 crash fears in the S&P 500 futures options market," *Journal of Econometrics* 94, 181-238.

Black, F., and. M. Scholes (1973), "The pricing of options and corporate liabilities," *Journal of Political Economy* 81, 637-54.

Bollerslev, T. (1986), "Generalized autoregressive conditional heteroscedasticity," *Journal of Econometrics* 31, 307-327.

Britten-Jones M., and A. Neuberger (2000), "Option price, implied volatility process, and stochastic volatility process," *Journal of Finance* 55, 839-866.

Chang, E. J., and B. M. Tabak (2007), "Are implied volatilities more informative? The

Brazilian real exchange rate case," *Applied Financial Economics* 17, 569-576.

Draper, P., and J. K. W. Fung (2002), "A Study of arbitrage efficiency between the FTSE-100 index futures and options contracts," *The Journal of Futures Markets* 22, 31-58.

Dumas, B., J. Fleming, and R. E. Whaley (1998), "Implied volatility functions: Empirical tests," *Journal of Finance* 53, 2059-2106.

Engle, R. F. (1982), "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kindom inflation," *Econometrica* 50, 987-1008.

Fair, R. C., and R. J. Shiller (1990), "Comparing information in forecasts from econometric models," *American Economic Review* 80, 375-389.

Ghysels, E., P. Santa-Clara, and R. Valkanov (2006), "Predicting volatility: Getting the most out of return data sampled at different frequencies," *Journal of Econometrics* 131, 59-95.

Jiang, G. J., and Y. S. Tian (2005), "The model-free implied volatility and its information content," *Review of Financial Studies* 18, 1305-1342.

Jorion, P. (1995), "Predicting volatility in foreign exchange market," *Journal of Finance* 50, pp. 507-528.

Fleming, J. (1998), "The quality of market volatility forecasts implied by S&P100 index options prices," *Journal of Empirical Finance* 5, 317-345.

Kroner, K. F. (1996), "Creating and using volatility forecasts," *Derivatives Quarterly*, 39-53.

Lee, J. H., and N. Nayar (1993), "A transactions data analysis of arbitrage between index options and index future," *The Journal of Futures Markets* 13, 889–902.

Newey, W. K., and K. D. West (1987), "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica* 55, 703-708.

Poon, S.-H., and C. W. J. Granger (2003), "Forecasting volatility in financial markets: A review," *Journal of Economic Literature* 41, 478-539.

Szakmary, A., E. Ors, J. K. Kim, and W. N. Davidson (2003), "The predictive power of implied volatility: evidence from 35 futures markets," *Journal of Banking & Finance* 27, 2151-2175.

Table 1 Summary statistics of the various volatility measures

Forecast* Horizon	Total No. of Obs.	Statistics	Realized Volatility	Black-Scholes Implied Volatility	Model-free Implied Volatility	Historical Volatility	GARCH(1,1)
H1	48	Mean	0.2096	0.2266	0.2212	0.2120	0.2254
		Std. Dev.	0.0729	0.0722	0.0615	0.0809	0.0676
		Maximum	0.4240	0.3773	0.3401	0.3909	0.3568
		Minimum	0.1160	0.1043	0.1157	0.0980	0.1194
H2	48	Mean	0.2014	0.2418	0.2212	0.2173	0.2291
		Std. Dev.	0.0675	0.0792	0.0653	0.0871	0.0719
		Maximum	0.3750	0.4119	0.3518	0.4110	0.4021
		Minimum	0.1023	0.1096	0.1142	0.0959	0.1165
H3	48	Mean	0.2043	0.2336	0.2100	0.2177	0.2300
		Std. Dev.	0.0680	0.0707	0.0530	0.0928	0.0761
		Maximum	0.3725	0.4151	0.3123	0.4518	0.4035
		Minimum	0.1066	0.1268	0.1282	0.0951	0.1152
H4	47	Mean	0.2063	0.2402	0.2090	0.2237	0.2396
		Std. Dev.	0.0658	0.0745	0.0590	0.0891	0.0819
		Maximum	0.3395	0.4375	0.3536	0.4540	0.4298
		Minimum	0.1025	0.1290	0.1274	0.0990	0.1160

Note: * The forecast horizons are based on the remaining time to maturity days of the TXO; these are weekly (H1), bi-weekly (H2), tri-weekly (H3) and monthly (H4) forecast horizons.

Table 2 Forecasting errors of the various forecasting methods[a]

Forecast Horizons	Total No. of Obs.	Forecast Error[b]	Black-Scholes Implied Volatility	Model-free Implied Volatility	Historical Volatility	GARCH(1,1)
H1	48	MAE	0.0406 (3)	0.0381 (1)	0.0434 (4)	0.0403 (2)
		RMSE	0.0534 (3)	0.0498 (1)	0.0596 (4)	0.0527 (2)
H2	48	MAE	0.0524 (4)	0.0404 (1)	0.0426 (3)	0.0421 (2)
		RMSE	0.0642 (4)	0.0485 (1)	0.0597 (3)	0.0539 (2)
H3	48	MAE	0.0441 (3)	0.0386 (1)	0.0487 (4)	0.0417 (2)
		RMSE	0.0529 (2)	0.0476 (1)	0.0673 (4)	0.0573 (3)
H4	47	MAE	0.0442 (4)	0.0367 (1)	0.0385 (2)	0.0421 (3)
		RMSE	0.0575 (3)	0.0468 (1)	0.0565 (2)	0.0587 (4)

Notes:

[a] The forecast horizons are based on the remaining time to maturity days of the TXO; these are weekly (H1), bi-weekly (H2), tri-weekly (H3) and monthly (H4) forecast horizons.

[b] MAE refers to mean absolute error; and RMSE refers to root mean squared error.

Table 3 Liquidity and relative performance of model-free implied volatility

Periods	$\hat{\alpha}$[b]		$\hat{\beta}$[b]		R^2
	Mean	t-statistic	Mean	t-statistic	
Whole Period	1.5490	4.1578**	-0.0031	-0.8256	0.0041
Period 1 (2002-2003)	2.3370	3.6158**	-0.0105	-1.7175	0.0421
Period 2 (2004-2005)	0.7585	5.7777**	0.0141	6.6624**	0.1541

Note:

[a] The results are from the estimated univariate regression model in Equation (11).

[b] ** indicates significance at the 1% level.

[c] The t-statistics are corrected for heteroskedasticity and serial correlation using the Newey and West (1987) variance-covariance estimator.

Table 4 Univariate regression results[a]

Forecast Horizon	Model	$\hat{\alpha}$[b]		$\hat{\beta}$[b]		R^2	χ^2[b]
		Mean	t-statistic[c]	Mean	t-statistic[c]		
H1	Black-Scholes Implied Volatility	0.0376	1.7776	0.7588	7.6311**	0.5644	8.5412*
	Model-free Implied Volatility	0.0136	0.5477	0.8860	7.4925**	0.5590	2.8251
	Historical Volatility	0.0761	4.1784**	0.6293	7.3865**	0.4883	19.0298**
	GARCH (1,1)	0.0296	1.5049	0.7986	8.5853	0.5486	7.4974*
H2	Black-Scholes Implied Volatility	0.0416	2.7754**	0.6607	11.7369**	0.6006	66.1576**
	Model-free Implied Volatility	0.0246	1.3835	0.7995	11.4818**	0.5975	15.8949**
	Historical Volatility	0.0759	4.4196**	0.5774	8.8635**	0.5552	58.0923**
	GARCH (1,1)	0.0341	1.8450	0.7300	9.5606**	0.6036	30.5079**
H3	Black-Scholes Implied Volatility	0.0259	1.4811	0.7638	11.4177**	0.6310	32.5041**
	Model-free Implied Volatility	0.0117	0.4358	0.9175	7.9081**	0.5109	1.1665
	Historical Volatility	0.0932	3.7263**	0.5106	4.9940**	0.4852	31.0156**
	GARCH (1,1)	0.0508	1.9069	0.6675	5.9075**	0.5573	20.9703**
H4	Black-Scholes Implied Volatility	0.0402	1.6657	0.6916	6.7456**	0.6139	25.3664**
	Model-free Implied Volatility	0.0387	1.3042	0.8017	6.1523**	0.5172	2.6382
	Historical Volatility	0.0749	3.8903**	0.5870	7.2939**	0.6320	34.7466**
	GARCH (1,1)	0.0518	2.3600*	0.6445	6.9978**	0.6446	40.2663**

Notes:

[a] The results shown are from the estimation of the univariate regression model in Equation (8); χ^2 is the Wald test statistic of the null hypothesis, H_0: $(\alpha, \beta = 0,1)$.

[b] ** indicates statistical significance at the 1% level, and * indicates statistical significance at the 5% level.

[c] The reported t-statistics are corrected for heteroskedasticity and serial correlation using the Newey and West (1987) variance-covariance estimator.

Table 5 Results of the encompassing regression on informational efficiency for Black-Scholes implied volatility [a]

Variables	Forecast Horizon [b] H1		H2		H3		H4		H4A [c]	
	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic
Panel A: Black-Scholes Implied Volatility and Historical Volatility										
$\hat{\alpha}$	0.0375	1.8166	0.0457	3.1699**	0.0229	1.1294	0.0530	2.1524*	0.0366	1.9830
$\hat{\beta}_1^{BS}$	0.5580	2.0676*	0.4590	3.4870**	0.8259	3.2649**	0.3113	1.5474	0.3641	1.8961
$\hat{\beta}_2^{HV}$	0.2151	0.9479	0.2058	1.5867	-0.0528	-0.2630	0.3509	2.2766*	0.3775	2.5293*
$\bar{R}^2$	0.5633		0.5980		0.6156		0.6383		0.6813	
$\chi^2_{(BS)}$	11.1993**		83.8110**		40.0010**		–		–	
$\chi^2_{(HV)}$	–		–		–		4.5273**		84.5886**	
Panel B: Black-Scholes Implied Volatility and GARCH (1,1)										
$\hat{\alpha}$	0.0176	0.8453	0.0290	1.7577	0.0255	1.5059	0.0394	1.7219	0.0237	1.3809
$\hat{\beta}_1^{BS}$	0.4431	1.9683	0.3405	2.0800*	0.6303	2.0283*	0.2825	1.5000	0.3649	2.0173*
$\hat{\beta}_2^{GARCH}$	0.4062	1.8497	0.3930	1.9018	0.1373	0.4296	0.4131	2.3813*	0.4046	2.3988*
$\bar{R}^2$	0.5912		0.6182		0.6190		0.6487		0.6832	
$\chi^2_{(BS)}$	10.4335*		80.3460**		32.0768**		–		43.6744**	
$\chi^2_{(GARCH)}$	–		–		–		36.5264**		59.6205**	

Note:

[a] The results shown are from the estimation of the encompassing regression model in Equation (9); $\chi^2_{(BS)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{BS} = 1$ and $\beta_2^{FV1} = 0$ ($FV1 = HV$, $GARCH$); $\chi^2_{(HV)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{BS} = 0$ and $\beta_2^{HV} = 1$; $\chi^2_{(GARCH)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{BS} = 0$ and $\beta_2^{GARCH} = 1$.

[b] The t-statistics are corrected for heteroskedasticity and serial correlations using the Newey and West (1987) variance-covariance estimator; ** indicates statistical significance at the 1% level, and * indicates statistical significance at the 5% level.

[c] The data on H4A is the same as for the H4 sample with the data for 20 May 2004 deleted; in other words, there are only 46 observations on H4A.

Table 6 Results of the encompassing regression on informational efficiency for model-free implied volatility[a]

Variables	Forecast Horizon[b]									
	H1		H2		H3		H4		H4A[c]	
	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic	Mean	t-statistic
Panel A: Model-free Implied Volatility and Historical Volatility										
$\hat{\alpha}$	0.0203	0.7636	0.0340	2.0429*	0.0331	1.1652	0.0725	2.4719*	0.0591	2.3804*
$\hat{\beta}_1^{MF}$	0.6383	2.1303*	0.5373	2.9610**	0.5639	1.9039	0.0321	0.1647	0.0558	0.3093
$\hat{\beta}_2^{HV}$	0.2269	1.0373	0.2233	1.4697	0.2430	1.1969	0.5679	5.4806**	0.6145	6.5574**
$\overline{R}^2$	0.5601		0.5992		0.5246		0.6154		0.6501	
$\chi^2_{(MF)}$	3.5106		22.0522**		3.1033		–		–	
$\chi^2_{(HV)}$	–		–		–		43.3008**		73.4311**	
Panel B: Model-free Implied Volatility and GARCH (1,1)										
$\hat{\alpha}$	0.0035	0.1470	0.0201	1.2432	0.0217	0.9456	0.0501	1.8631	0.0369	1.5557
$\hat{\beta}_1^{MF}$	0.5065	2.0371*	0.4036	2.0940*	0.3845	1.3731	0.0314	0.1595	0.0859	0.4817
$\hat{\beta}_2^{GARCH}$	0.4173	2.0025	0.4018	1.9540	0.4431	1.8662	0.6244	4.7435**	0.6390	5.2481**
$\overline{R}^2$	0.5887		0.6175		0.5655		0.6287		0.6515	
$\chi^2_{(MF)}$	5.3358		23.3168**		5.3866		–		–	
$\chi^2_{(GARCH)}$	–		–		–		45.6286**		50.0438**	

Note:

[a] The results shown are from the estimation of the encompassing regression model in Equation (10); $\chi^2_{(MF)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{MF} = 1$ and $\beta_2^{FV1} = 0$ ($FV1 = HV, GARCH$); $\chi^2_{(HV)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{MF} = 0$ and $\beta_2^{HV} = 1$; $\chi^2_{(GARCH)}$ is the Wald test statistic of the null hypothesis, H_0: $\beta_1^{MF} = 0$ and $\beta_2^{GARCH} = 1$.

[b] The t-statistics are corrected for heteroskedasticity and serial correlations using the Newey and West (1987) variance-covariance estimator; ** indicates statistical significance at the 1% level, and * indicates statistical significance at the 5% level.

[c] The data on H4A is the same as for the H4 sample with the data for 20 May 2004 deleted; in other words, there are only 46 observations on H4A.

Appendix A

Table A-1 *Realized volatility and forecast volatility of various models, by monthly (H4) forecast horizons*

Date	Month	Realized Volatility	Black-Scholes Implied Volatility	Model-free Implied Volatility	Historical Volatility	GARCH	Ratio of Absolute Error	Spread
17 January 2002	February 2002	0.2948	0.3762	0.2901	0.3255	0.2930	0.0580	32.2500
21 February 2002	March	0.2842	0.3166	0.2498	0.2797	0.2873	1.0643	93.7500
21 March 2002	April	0.2263	0.2649	0.2098	0.2799	0.2824	0.4284	30.1667
18 April 2002	May	0.2601	0.2287	0.1398	0.1793	0.2424	3.8261	25.7143
23 May 2002	June	0.2539	0.2723	0.2505	0.3287	0.3314	0.1894	18.6133
20 June 2002	July	0.3016	0.2969	0.2559	0.2553	0.2769	9.7377	13.6267
25 July 2002	August	0.3163	0.3032	0.3011	0.3183	0.3134	1.1597	43.3667
22 August 2002	September	0.2289	0.3178	0.2952	0.3451	0.2629	0.7457	25.1045
19 September 2002	October	0.3068	0.3180	0.2767	0.2983	0.3693	2.6905	9.5857
24 October 2002	November	0.2798	0.3286	0.3063	0.3764	0.4153	0.5426	51.7333
21 November 2002	December	0.2424	0.3168	0.2841	0.2646	0.2865	0.5612	19.8286
19 December 2002	January 2003	0.2414	0.2289	0.2016	0.1984	0.2328	3.1914	19.8769
16 January 2003	February	0.2666	0.2079	0.1954	0.2305	0.2643	1.2137	60.4167
20 February 2003	March	0.2842	0.2776	0.2479	0.2975	0.3303	5.5175	74.1739
20 March 2003	April	0.1956	0.3296	0.2827	0.2865	0.3261	0.6500	14.5000
23 April 2003	May	0.2485	0.2437	0.2074	0.2507	0.2766	8.6688	8.4000
22 May 2003	June	0.2354	0.2433	0.2185	0.2658	0.2433	2.1078	7.1105
19 June 2003	July	0.2195	0.2752	0.1875	0.2158	0.2466	0.5736	36.3571
24 July 2003	August	0.2003	0.2494	0.2069	0.2436	0.2537	0.1334	64.8421
21 August 2003	September	0.1893	0.2568	0.1978	0.1979	0.2427	0.1247	87.9500
18 September 2003	October	0.1669	0.2031	0.1742	0.1711	0.2232	0.2012	16.7727
23 October 2003	November	0.1715	0.1915	0.1711	0.1643	0.2005	0.0200	18.6250
20 November 2003	December	0.1605	0.2008	0.1803	0.1528	0.1886	0.4908	251.0769

Table A-1 (Contd.)

Date	Month	Realized Volatility	Black-Scholes Implied Volatility	Model-free Implied Volatility	Historical Volatility	GARCH	Ratio of Absolute Error	Spread
22 December 2003	January 2004	0.1555	0.1467	0.1435	0.1666	0.1785	1.3666	2.5100
28 January 2004	February	0.1696	0.1648	0.1524	0.1461	0.1663	3.6258	19.5417
19 February 2004	March	0.2068	0.1778	0.1432	0.1532	0.1549	2.1923	102.0800
25 March 2004	April	0.2177	0.2826	0.2781	0.3452	0.3658	0.9310	6.6235
22 April 2004	May	0.3376	0.2841	0.2311	0.2584	0.2563	1.9895	22.0000
20 May 2004	June	0.2616	0.4375	0.3536	0.4540	0.4298	0.5229	16.6500
24 June 2004	July	0.2044	0.3279	0.2752	0.3202	0.3307	0.5735	6.0625
22 July 2004	August	0.1735	0.2885	0.2511	0.2029	0.2568	0.6744	2.3500
19 August 2004	September	0.1723	0.2514	0.2263	0.1829	0.1921	0.6826	5.8045
22 September 2004	October	0.1626	0.1976	0.1768	0.1670	0.1914	0.4068	14.6250
21 October 2004	November	0.1650	0.2135	0.2011	0.1640	0.1886	0.7436	20.5000
18 November 2004	December	0.1667	0.2278	0.2088	0.1608	0.1848	0.6887	16.4640
23 December 2004	January 2005	0.1371	0.1870	0.1689	0.1092	0.1546	0.6360	16.9071
17 February 2005	March	0.1262	0.1390	0.1313	0.1301	0.1482	0.4009	29.7053
23 March 2005	April	0.1358	0.1290	0.1276	0.1036	0.1321	1.2076	4.5133
21 April 2005	May	0.1245	0.1587	0.1538	0.1584	0.1800	0.8566	2.4313
19 May 2005	June	0.1025	0.1471	0.1415	0.1251	0.1465	0.8747	4.4941
22 June 2005	July	0.1208	0.1472	0.1340	0.1128	0.1219	0.4996	12.7059
21 July 2005	August	0.1191	0.1685	0.1590	0.0991	0.1160	0.8072	15.9824
24 August 2005	September	0.1137	0.1304	0.1274	0.1269	0.1270	0.8210	11.6889
22 September 2005	October	0.1300	0.1513	0.1472	0.1253	0.1268	0.8040	2.9000
20 October 2005	November	0.1517	0.1453	0.1444	0.1752	0.1761	1.1422	6.1625
24 November 2005	December	0.1253	0.1680	0.1528	0.1637	0.1601	0.6437	16.6316