Load And Friction-loss Performances of Compensated Elastohydrostatic Circular Step Thrust Bearings

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Abstract

To enable the lubricant supply to be easier and reduce bearing wear and scoring, this study is mainly concerned with the elastohydrostatic lubrication of orifice and capillarycompensated circular step thrust bearings by using an elastic upper pad on the runner surface. The Reynolds-type equation governing the film pressure is drived to modify the local film shape. The elastohydrostatic film pressure, load-carrying capacity and viscous dissipation loss are then evaluated. To show the effect of elastic material layer, the above bearing characteristics are presented with various values of nondimensional elastohydrostatic parameter and inner step radius ratio.

Keywords:elastohydrostatic thrust bearings, compensated restrictors, load performance, viscous dissipation loss.

補償性彈性靜壓盤狀步階推力軸承之負荷與摩擦損耗性能

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摘　要

為使潤滑劑供應順暢及降低軸承之磨損，本文探討利用一撓性襯墊黏著於轉子端面後，補償性盤狀步階推力軸承之彈性靜壓潤滑性能。當彈性膜壓從雷諾型方程解出後，接著用以計算負荷與摩擦損耗性能。為能顯示使用撓性材質襯墊後之軸承特性，上述彈性靜壓軸承數據將與剛性靜壓軸承數據做一比較。

關鍵字：彈性靜壓盤狀步階推力軸承，補償元件，負荷性能，黏性耗散損失

Introduction

Hydrostatic lubricated bearings play an important role in engineering practice because of their low starting friction, small viscous running dissipation, and high load-carrying capacity. Industrial applications are commonly observed in aircraft engines, machine tools, radar antennas, magnetic elements supports, gyroscope gimbals, dynamometers, and turbobine-generartor units, etc. Fuller[1] studied the hydrostatic characteristics of circular step thrust bearings in early days. Many articles dealing with these kinds of bearings have been presented by considering different operating situations, such as the influence of centripetal accelerations of Dowson[2], the optimization of bearing stiffness of Ling[3], the minimum power and low temperature rise of Rowe et al. [4], the temperature and rotational inertia effects of Ting and Mayer[5], the misaligned effect of Safar[6,7], the dynamic stiffness and damping characteristics of Ghosh and Majumdar[8], and the turbulent lubrication of khalil and Ismail[9]. All the above studies of hydrostatic circular thrust mechanism focused on the performance characteristics of bearings with rigid material. To enable the lubricant supply to be easier and reduce bearing wear and scoring, a layer of flexible materials is usually applied on the bearing surface. In such situations, further studies of the film profile and bearing characteristics show physically significant. These kinds of elastohydrostatic performance of thrust bearings have been analyzed by Dowson[10] and by Singh et al. (1982). However, their studies concentrated upon the performance characteristics of circular plate bearings with a uncompensated restrictor. For industrial practice a further investigation of elastohydrostatic characteristics for circular step thrust bearings with compensated restrictors is needed. Although Sinhasan et al.[12] investigated the problem of elastohydrostatic lubrication of capillary compensated bearings, they used the flexible annular pad on the stator.

By applying an elastic upper pad on the runner surface, the present paper is mainly concerned with the elastohydrostatic characteristics of circular step thrust bearings with orifice and capillary compensations. The Reynolds equation governing the pressure is drived to modify the local film shape. The recess boundary pressure is determined from the recess flow continuity equation. The bearing film pressure, load-carrying capacity, and dissipation loss are examined with various values of nondimensional elastohydrostatic parameter and inner step radius ratio.

Formulation

Consider the laminar flow of an incompressible Newtonian lubricant in a hydrostatic circular step thrust bearing as shown in Fig. 1. The lubricant from an externally pressurized source through a restrictor is fed into the bearing recess of radius r_i. The upper pad is lined with flexible material with thickness t_h. The runner is rotating about z axis with speed Ω. With the speed being low, the rotational inertia effect is then neglected. Under the usual assumptions of thin film lubrication theory the equations of motion governing the flow of an incompressible fluid in cylindrical polar coordinates are given by

$$0 = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 u}{\partial z^2} \tag{1}$$

$$0 = \mu \frac{\partial^2 v}{\partial z^2} \tag{2}$$

$$0 = \frac{\partial p}{\partial z} \tag{3}$$

$$\frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Integrate equations (1) and (2) with boundary conditions at the surfaces of step pad and upper elastic pad:

$$u(r,0) = 0, \qquad u(r,h) = 0 \tag{5}$$

$$v(r,0) = 0, \qquad v(r,h) = r\Omega \tag{6}$$

Where h denotes the local film height. The radial and tangential velocities are then obtained.

$$u(r,z) = \frac{1}{2\mu}\frac{dp}{dr}(z^2 - hz) \tag{7}$$

$$v(r,z) = r\Omega\frac{z}{h} \tag{8}$$

Substitute equation (7) and (8) into continuity equation (4) and integrate with respect to z with boundary conditions of $w(r,z)$:

$$w(r,0) = w(r,h) = 0 \tag{9}$$

As a result, the Reynolds-type equation governing the film pressure is derived.

$$\frac{1}{r}\frac{d}{dr}\left(h^3 r\frac{dp}{dr}\right) = 0 \tag{10}$$

Elastohydrostatic Film Thickness. With h_0 denoting the sonstant film thickness in the unstressed case, the elastohydrostatic film thickness under stressed case is modified by the local compression of the flexible upper pad, δ.

$$h = h_0 + \delta \tag{11}$$

Assume that the elastic compression is determined by the local pressure and that the thin flexible pad is firmly attached to the runner surface, the local compression can be obtained by Hooke's law.

$$\delta = \frac{pt_h}{E'} \tag{12}$$

Where E' is an equivalent elastic modulus based upon the assumption of uniform pressure and rigid lateral constraint

$$\frac{1}{E'} = \frac{1}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right] \tag{13}$$

Dimensionless Reynolds-type Equation. Introducing the dimensionless variables and parameters,

$$r^* = \frac{r}{r_0}, \quad h^* = \frac{h}{h_0}, \quad p^* = \frac{p}{p_s}, \quad M^* = \frac{t_h}{E'} \cdot \frac{p_s}{h_0} \tag{14}$$

One obtains the modified Reynolds equations written in a nondimensional form.

$$\frac{1}{r^*}\frac{d}{dr^*}\left(h^{*3} r^* \frac{dp^*}{dr^*}\right) = 0 \tag{15}$$

Where the elastohydrostatic film height depends the values of elasohydrostatic bearing parameter M and film pressure p^*.

$$h^* = 1 + M^* p^* \tag{16}$$

Load Performance and Dissipation Loss

The boundary conditions for film pressure are

$$p^* = p_r^* \quad \text{at} \quad r^* = \alpha \tag{17a}$$

$$p^* = 0 \quad \text{at} \quad r^* = 1 \tag{17b}$$

Where P_r^* is the recess boundary pressure. The value of P_r^* can be obtained from the continuity of flow rate through the bearing recess (see **Recess Boundary pressure**). Solving the Reynolds equation for pressure with the above conditions gives the elastohydrostatic film pressure distribution.

$$p^* = \frac{1}{M^*}\left\{\left[1+\frac{\left(1+M^* p_r^*\right)^4-1}{\ln\alpha}\ln r^*\right]^{1/4}-1\right\} \tag{18}$$

Taking the limit $M^* \to 0$, equation (18) yields the film pressure distribution for rigid bearings (Hamrock, 1994).

$$p^*(rigid) = p_r^* \frac{\ln r^*}{\ln\alpha} \tag{19}$$

The load capacity is calculated by integrating the film pressure ovre the upper elastic pad.

$$W = \pi r_i^2 p_r + 2\pi \int_{r=r_i}^{r=r_0} p(r)rdr \tag{20}$$

Expressing in dimensionless form, one has

$$W^* = \frac{W}{A_p p_r} = \alpha^2 - \frac{1-\alpha^2}{M^* p_r^*} + \frac{2}{M^* p_r^*} Gw(\alpha, M^*, p_r^*) \tag{21}$$

where A_p denotes the total projected area and

$$Gw(\alpha, M^*, p_r^*) = \int_{r^*=\alpha}^{r^*=1}\left[1+\frac{(1+M^* p_r^*)^4-1}{\ln\alpha}\ln r^*\right]^{1/4} r^* dr^* . \tag{22}$$

Taking the limit $M^* \to 0$, equation (21) yields the load coefficient for rigid bearings Hamrock, [13]

$$W^*(rigid) = \frac{\alpha^2-1}{2\ln\alpha} \tag{23}$$

To analyze the effect of elastic pad on load performance, the increasing rate are defined as

$$Cw = \frac{W^*(elasto) - W^*(rigid)}{W^*(rigid)} \times 100 \tag{24}$$

The friction torque due to viscous dissipation is

$$T = \int_{r=0}^{r=r_0} \int_{\theta=0}^{\theta=2\pi} \left[\mu \frac{\partial v}{\partial z} \right]_{z=h} r^2 \, d\theta dr \tag{25}$$

The friction power H_f is

$$H_f = T\Omega \tag{26}$$

Expressing in dimensionless form gives

$$H_f^* = \frac{2\pi S^2}{p_r^{*2} W^{*2}} \left[\frac{\alpha^4}{4(1+M^* p_r^*)} + G_f(\alpha, M^* p_r^*) \right] \tag{27}$$

Where

$$S = \frac{\mu \Omega r_0^2}{p_s h_0^2} \tag{28}$$

$$G_f(\alpha, M^*, p_r^*) = \int_{r^*=\alpha}^{r^*=1} \frac{r^{*3}}{\left[1 + \frac{(1+M^* p_r^*)^4 - 1}{\ln \alpha} \ln r^* \right]^{1/4}} dr^* \tag{29}$$

Taking the limit $M \to 0$, equation (27) yields the case for a rigid bearing (Hamrock, 1994).

$$H_f^*(rigid) = \frac{\pi S^2}{2 p_r^{*2} W^{*2}} \tag{30}$$

To analyze the effect of elastic pad on friction loss, the decreasing rate are defined as

$$G_f = -\frac{H_f^*(elasto) - H_f^*(rigid)}{H_f^*(rigid)} \times 100 \tag{31}$$

Recess Boundary Pressure

To obtain the value of recess boundary pressure P_r^* ,the condition of flow continuity through the bearing recess is utilized. The lubricant flow rate is evaluated by integrating the velocity field.

$$Q = \int_{z=0}^{z=h}\int_{\theta=0}^{\theta=2\pi} u\Big|_{r=r_i} r_i d\theta dz = \frac{p_s h_0^3 \pi\left[1-(1+M^* p_r^*)^4\right]}{2\mu M^*(1-p_r^*)^{1/2}\ln\alpha} \tag{32}$$

Since the lubricant flow fed to the bearing sill is controlled by the restrictor, one has also

$$Q = \frac{\pi d_0^2}{4} C_d \left[\frac{2}{\rho}(p_s - p_r)\right]^{1/2} \quad \text{(orifice)} \tag{33a}$$

$$Q = \frac{\pi d_c^4}{128\mu l_c}(p_s - p_r) \quad \text{(capillary)} \tag{33b}$$

Introducing the restrictor parameters δ_0 and δ_c

$$\delta_o = \frac{3(2)^{1/2}\pi d_0^2 C_d \mu}{(\rho p_s)^{1/2} h_0^3}, \delta_c = \frac{3\pi d_c^4}{32 h_0^3 l_c} \tag{34}$$

And combining equations (32) and (33) lead to the following relations.

$$\pi(1+M^* p_r^*)^4 - 2\delta_0 M^* \ln\alpha \cdot (1-p_r^*)^{1/2} - \pi = 0 \quad \text{(orifice)} \tag{35a}$$

$$\pi(1+M^* p_r^*)^4 - 2\delta_c M^* \ln\alpha \cdot (1-p_r^*) - \pi = 0 \quad \text{(capillary)} \tag{35b}$$

With the values of δ_0 or δ_c , α and M^* given, the recess pressure can be determined from equations (35a) or (35b) and then is applied to be the boundary condition of film pressure.

Results and Discussion

By using an elastic upper pad on the runner surface, the elastohydrostatic lubrication of circular step thrust bearings is theoretically studied. The recess boundary pressure is determined from the recess flow continuity equation. When the values of restrictor parameter (δ_0 or δ_c), radius ratio (α) and elastohydrostatic parameter (M^*) are specified, the dimensionless recess pressure (p_r^*)can be obtained from equations (35a) or (35b), and then applied to calculate the film pressure in equation (18). With the elastohydrostatic film pressure solved from the modified Reynolds equation, the bearing load and viscous dissipation loss are thus obtained according to equations (21) and (27). Since it is concerned with the steady performance of bearings, for convenience the elastohydrostatic bearing characteristics presented in Figs. 2~7 are evaluated under the recess pressure $p_r^* = 0.7$

Figure 2 shows the variation of dimensionless elastohydrostatic film pressure p^* versus dimensionless radius r^* for various values of elastohydrostatic parameter M^* and inner radius ratio α . The film pressure increases for a bearing with elastic upper pad as compared to the rigid bearing. It is also found that the elastic upper pad affects the film pressure especially for smaller value of α . Figure 3 describes the variation of dimensionless elastohydrostatic film profile h^* versus dimensionless radius r^* for different values of elastohydrostatic parameter M^*. Since the higher pressure is generated for the elastohydrastatic bearings (M=0.15, 0.25), the film profile is thus found to be higher than the rigid bearing. Figure 4 presents the variation of dimensionless load coefficient W^*as a function of dimensionless inner radius ratio α for different values of elastohydrostatic parameter M^*. It is found that the presence of elastic pad increases the load-carrying capacity of bearings. Moreover, the increment is increased as the value of α decreases. This results can be realized that since the increment of film pressure is enlarged, the integrated load is similarly affected. Figure 5 shows the variation of increasing rate of load C_w with elastohydrostatic parameter M^* for various values of inner radius ratio α. The percentage rate of increase of load increases with the increase of M^*. It is also observed that the rate of increase is enhanced by a smaller value of α. Figure 6 presents the variation of dimensionless friction power H_f^* as a function of dimensionless inner radius ration α for different values of elastohydrostatic parameter M^*. The value of speed parameter S decribes the rotational effect of rotor with low speed. The use of elastic pad is observed to decrease the viscous dissipation power H_f^*. Figure 7 shows the variation of decreasing rate of friction power C_f with elastohydrostatic parameter M^* for different values of inner radius ratio α. The percentage rates of decrease of friction power increases with the increase of M. Moreover the rate of decrease friction power increases with a smaller value of α.

Conclusions

To enable the lubricant supply to be easier and reduce bearing wear and scoring, the problem of elastohydrostatic lubrication of circular step bearings with orifice and capillary compensations is theoretically studied by the use of an elastic upper pad attached to the runner end surface. The recess boundary pressure is determined from the recess flow continuity equation. The Reynolds-type equation governing the pressure is drived to modify the local film shape. The elastohydrostatic film pressure is solved and applied to calculate the bearing characteristics. According to the results obtained, the effect of elastic pad increases the elastohydrostatic film pressure, and provides a load-enhanced capacity and a friction-reduced power. Moreover, the influence of elastohydrostatic parameter M^* on the bearing characteristics is more pronounced for small values of inner radius ratio α.

Based upon the present results for steady performance of elastohydeostatic lubrication of bearings, a further study on dynamic characteristics will be presented in the future.

Nomenclature

A_p	total projected pad area, $A_p = \pi r_o^2$, m^2
C_d	coefficient of discharge of orifice (nondimensional)
C_f	decreasing rate of dissipation power (nondimensional)
C_w	increasing rate of load-carrying capacity (nondimensional)
d_c	capillary diameter, m
d_o	orifice diameter, m
E	modulus of elasticity of the flexible pad, N/m^2
E'	equivalent elastic modulus, N/m^2
h	elastohydrostatic film thickness, $h=h_o+\delta$, m
h_o	film thickness in the unstressed case, m
h^*	dimensionless film thickness, $h^* = \frac{h}{h_o}$
l_c	length of capillary, m^2
M^*	dimensionless elastohydrostatic parameter, $M^*=\frac{p_s t_h}{E' h_o}$
P	elastohydrostatic film pressure, N/m^2
p_s	supply pressure, m^*
p^*	dimensionless elastohydrostatic film pressure, $p^*=\frac{p}{p_s}$
p_r^*	dimensionless elastohydrostatic film pressure, $p_r^*=\frac{p_r}{p_s}$
Q	lubricant flow rate, m^3/s
r, θ, z	cylindrical polar coordinates
r_i , r_o	inner radius and outer radius of the step pad, m
r^*	dimensionless radius, $r^*=\frac{r}{r_o}$
S	dimensionless speed parameter, $S=\frac{\mu \Omega r_o^2}{p_s h_o^2}$

μ , ν , w	velocity of lubricant in r, θ and z directions, m/s
W	load-carrying capacity, N
W^*	dimensinless load coefficient, $w^* = \frac{w}{A_p p_r}$
α	dimensionless inner radius ratio, $\alpha = \frac{r_i}{r_o}$
δ	local displacement of elastic upper pad, $\delta = \frac{p t_h}{E'}$, m
δ_c	dimensionless capillary parameter, $\delta_c = \frac{3\pi d_c^3}{32 h_o^3 l_c}$
δ_o	dimensionless orifice parameter, $\delta_O = \frac{3(2)^{1/2} \pi d_o^2 C_{du}}{(\rho p_s)^{1/2} h_o^3}$
μ	lubricant viscosity, $N \cdot s/m^2$
ν	poisson's ratio (nondimensional)
ρ	lubricant density, $N.\ s/m^2$
Ω	rotation speed of runner, rad/s

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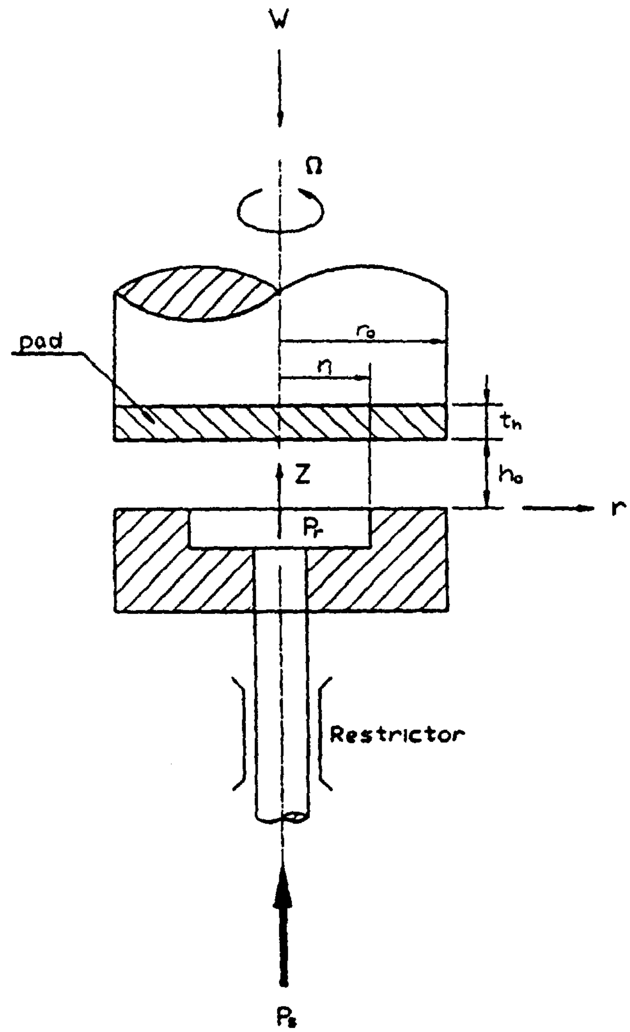

Fig. 1 Compensated elastohydrostatic circular step thrust bearing.

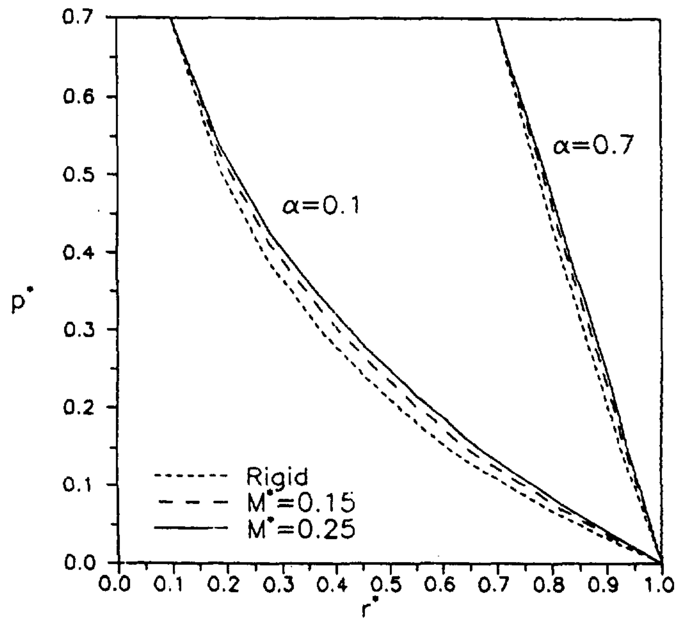

Fig. 2 Dimensionless elastohydrostatic pressure p^* versus r^* for various values of M^* and α

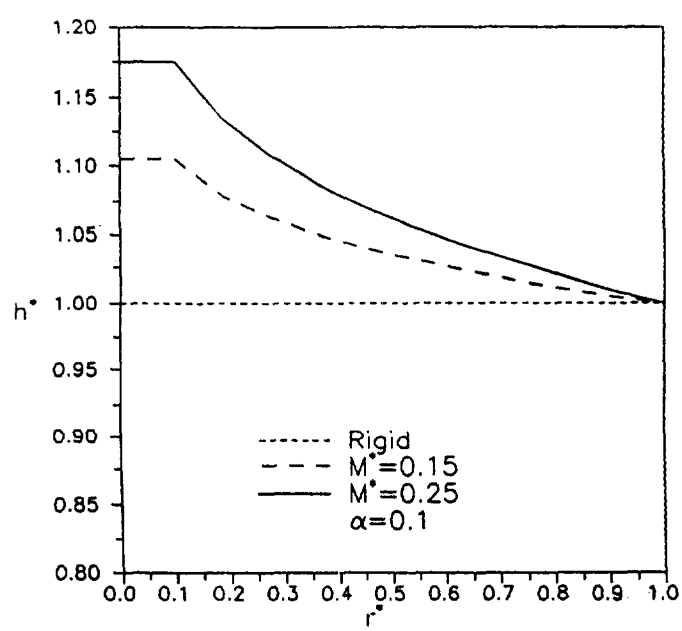

Fig. 3 Dimensionless film profile h^* versus r^* for various values of M^*.

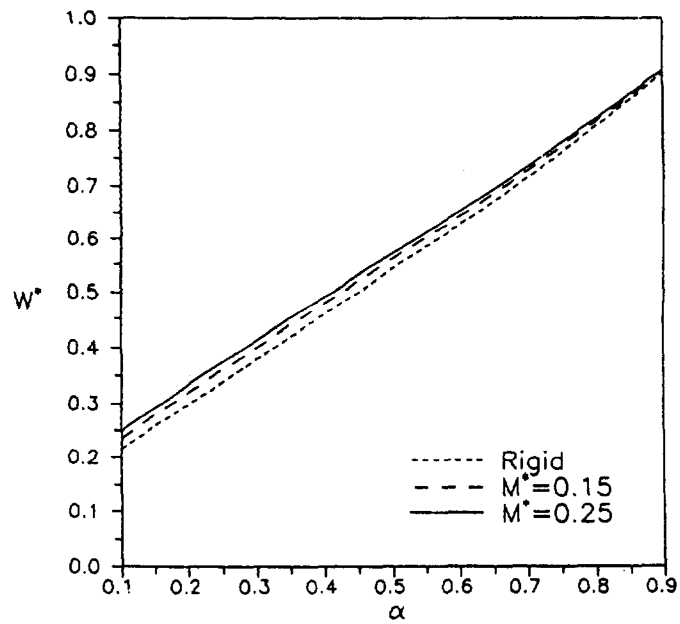

Fig. 4 Dimensionless load coefficient W^* as a function of α for different values of M^*

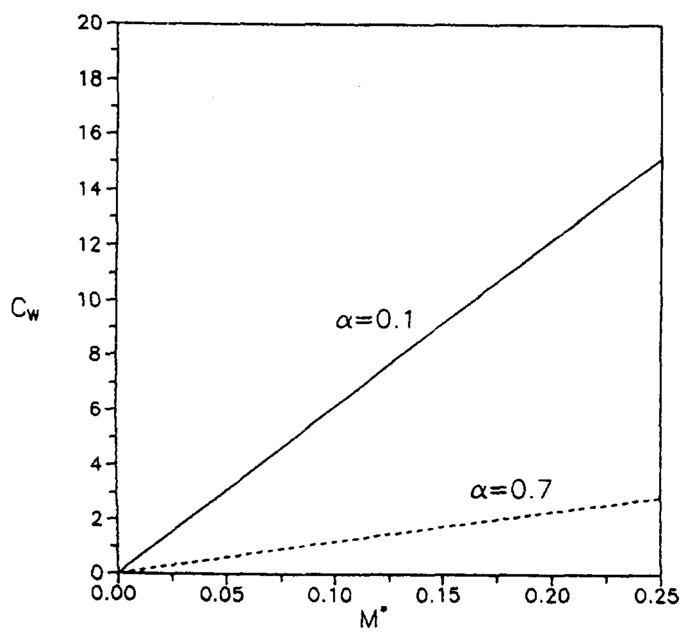

Fig. 5 Variation of load-increasing rate C_w with M^* for various values of α

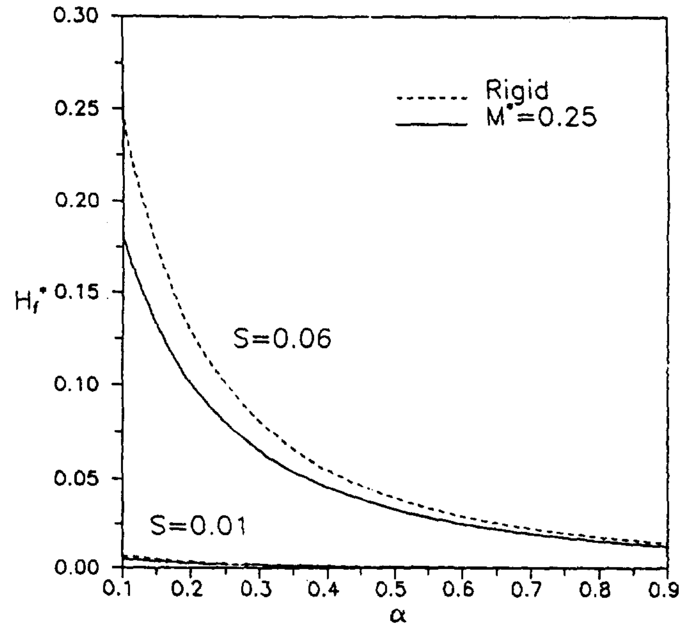

Fig. 6 Dimensionless friction power H_f^* as a function of α for different values of M

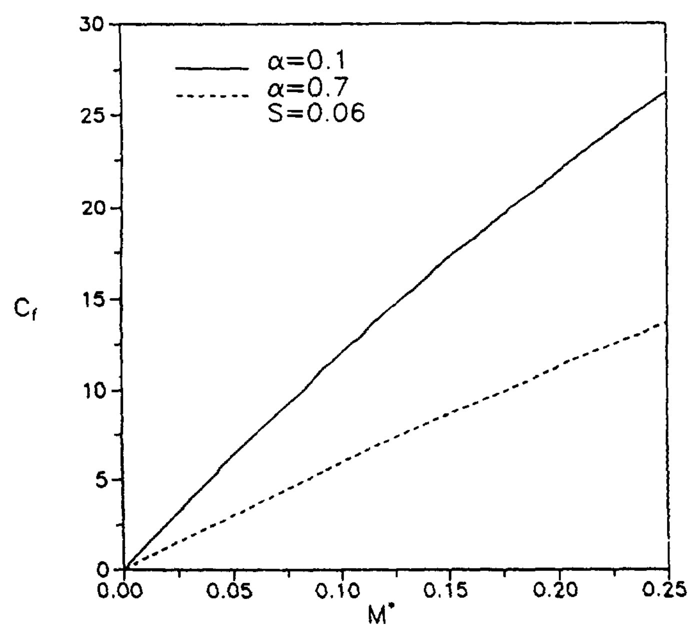

Fig. 7 Variation of friction power-decreasing rate C_f with M for various values of α