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USING THE INVERSE METHOD FOR ESTIMATING EXTERNAL FORCES FROM STRUCTURAL SYSTEM

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ABSTRACT

This paper investigates to predict the non-measurable displacement and forces of the structural system in time histories from accelerometer simulated records. It deals specifically with the situation where the external forces are unknown. The accelerometer and beam (truss) were modeled as a structural system in order to get a system governing equation. The matrix representation of governing equation of the structural system was constructed by the finite element method. Then the matrix representation was transformed to state equation and measurement equation of the Kalman filter by using the average acceleration method. The Kalman filter was adopted to predict, innovate and filter for the estimable displacement without external forces. Then using the recursive least square algorithm to decrease error between measurement and estimable of the external forces, therefore this paper finally got the estimable external forces. The accuracy of the current method is demonstrated by the numerical simulations for the external forces applied at the specific location of the beam (truss) in time histories.

INTRODUCTION

In the measurement of the actual phenomena, the observed data often suffer from losses or distortions due to the limited dynamic range of the measurement instruments. The fluctuation pattern of a non-Gaussian type is complicated because of physical, environmental, and psychological factors. In these measurements, the fluctuations of a random signal are inevitably contaminated by the external noise of the arbitrary distribution type. Furthermore, since all measuring equipments have a limited dynamic range, therefore the designer does his best to eliminate all the obvious noise-producing mechanisms. The Kalman filter was proposed in 1960[1], and has evoked much interest among engineers and scientists. The Kalman filtering technique with its recursive structure has been applied to the processing noisy measurement problems. For handling the process noisy problems, Efe *et al.*[2] used two different adaptive Kalman filters design to track targets expected to perform varying turn maneuvers.

It is a very important task in structural design for determination of excitation forces, but the task is not always practicable to directly measure the excitation forces, for example excitations of wind, seismic, explosion and shock. In particular, for impulsive or impact resistance and reliability of structural material in the modern day, they need acute analytical studies, because of their remarkable susceptibility to impact damage, and sometimes direct measurements of the impulsive loads are not feasible. So an indirect estimation for the excitation forces is frequently employed. For the input forces estimation, it is the process of determining the applied loadings from measurements of the system responses. Some techniques have handled the inverse problem during force estimation. Lee and Park [3] adopted the characteristics of the force determination error in structural dynamic systems. They derive the major factors affecting the force determination error and propose a regularization procedure to reduce the error. For biomechanical studies of locomotion, Bogert and Nigg

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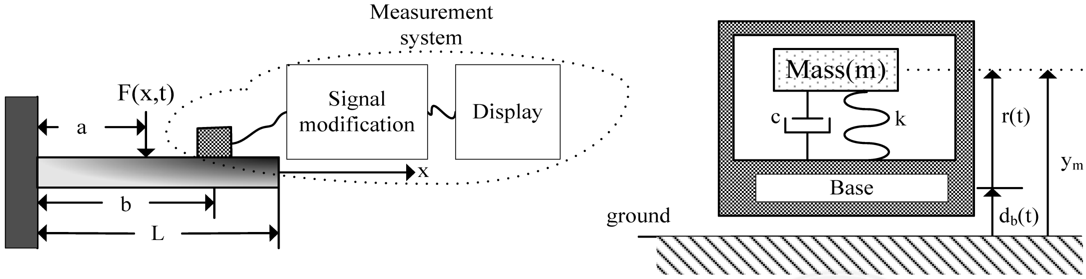
[4] proposed a method for inverse dynamic analysis using accelerometers to replace kinematics of the body segments with measurement of external forces to obtain the resultant force and moment. Tuan *et al.* [5,6] proposed an on-line input estimation method includes Kalman filter with a least square algorithm to estimate heat flux from measured temperature. They applied the algorithm to one-dimensional and two-dimensional heat conduction problems. However, they used finite difference method to set the state equation of Kalman filter. Furthermore, Ma *et al.* used Finite Element Method (FEM) to construct the system state equations of the beam structures, and then used the on-line input estimation method includes Kalman filter with a least square algorithm to estimate the unknown excitation forces from beam vibration modes [7,8]. Deng *et al.* [9] used FEM to construct the state equation and measurement equation of the Kalman filter for a cantilever beam with a uniform load. However, their researches didn't include the measurement sensors, and assumed the displacement is known.

For the measurement sensors, the accelerometers are commonly used in the measurement of engineering fields. Owing to accelerometers advantages, i.e. high accuracy, wide-band frequency and dynamic ranges, small size, lightweight, and ease of installation, the accelerometers are the preferred motion sensors for most shock and vibrations monitoring applications [10]. Link and Martens [11] design a one-degree-of-freedom system model for an accelerometer. This model set, in conjunction with the determination of the accelerometer input signal by laser interferometer and appropriate signal processing methods, allows computing the input-output behavior of accelerometers using known identification algorithms. The piezoelectric accelerometers were modeled as a one-degree-of-freedom mass-damper-spring system, too. The principle and design of the sensor have been presented for its robustness and low cost [12]. Even some scholars utilized Neural Networks as the estimation approach for the estimation and explicit formulation of available rotation capacity of wide flange beams [13].

Based on the above developments, recent methods, which describe the de-convolution of the unknown system (instrument), apply the total least squares (TLS) method [14]. In addition, in experiments, Da et al. [15] develop an assessment methodology based on vibrations tests and finite element analysis to predict the fatigue life of electronic components under random vibration loading. This research simulates an accelerometer to measure the acceleration of beam to estimate the unknown external forces. The measured accelerometer input signals are simulated from the forward solution with statistical noises. The accelerometer was modeled as a one-degree-of- freedom mass-damper-spring system. This system then has been added to the beam structural system as one integrated system. We use Kalman filter with least square algorithm to estimate the external forces of this integrated system from simulated measuring acceleration of the accelerometer directly.

In this section, the FEM is applied to derive a mass matrix, stiffness matrix and external forces vector of the beam structure. Then we construct the system's state equation of the Kalman filter, and establish an estimation scheme to determine the unknown excitation force from disturbing signals of the input-output data from an accelerometer from Fig. 1(a).

The accelerometer is considered to be a linear dynamic system and can be described by a one-degree-of-freedom piezoelectric system. From Fig. 1(b), the piezoelectric element is modeled by a spring with a stiffness constant k and viscous damping c. The absolute position of the accelerometer is d_b(t) and its acceleration is $\gamma(t) = d^2 d_b(t)/dt^2$. Then the absolute position of the mass M is $y_m(t)$ and the



(a) Accelerometer on cantilever beam (b) Enlargement of Accelerometer

Fig. 1 Concentrated load $F(x,t)$ on a cantilever beam with a measurement system relative position between the mass and the body, $r(t)$, is given by [12]

$$r(t) = y_m(t) - d_b(t) \tag{1}$$

Newton's second law applied to the mass m for little variations and projected to the motion axis gives the following equation:

$$m\frac{d^2 y_m}{dt^2} = -c\,\frac{dr}{dt} - k\,r(t) \tag{2}$$

Then, we replace y_m with equation (1), and it can get

$$m\gamma(t) = m\frac{d^2 d_b}{dt^2} = -(m\frac{d^2 r}{dt^2} + c\,\frac{dr}{dt} - k\,r(t)) \tag{3}$$

To integrate an accelerometer with the cantilever beam structural system, we want to predict the unknown external force.

1. State equations of the system

From Fig. 1, the lateral vibration of the Euler-Bernoulli beam structure with an accelerometer is governed by the equation

$$\bar{\rho}\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = F(a,t) - m\frac{\partial^2 r}{\partial t^2}\delta(x-b) - c\frac{\partial r}{\partial t}\delta(x-b) - kr\,\delta(x-b) \tag{4}$$

where $y(x,t)$ is the transverse displacement of the beam, and x denotes the spatial axis along the beam axis and t is time. The mass density per length $\bar{\rho}$ is a constant, EI is the beam rigidity, and $F(x,t)$ is the externally applied force [16]. Here a and b are the location of the force and accelerometer on this beam respectively. $\delta(.)$ is the Dirac delta function.

We apply the Galerkin's method to Equation (4) to develop the finite element formulation [12] and the corresponding matrix equations. Therefore, the matrix equation for this dynamic beam can be given by

$$[M]\{\ddot{d}\} + [K]\{d\} = \{F(a,t) - m\frac{\partial^2 r}{\partial t^2}\delta(x-b) - c\frac{\partial r}{\partial t}\delta(x-b) - kr\delta(x-b)\} \tag{5}$$

where M is element n×n mass matrix, K denotes element n×n stiffness matrix, and F represents the external force of the n×1 load vector.

Assuming the accelerometer output data $d_b = \rho\, r(t)$, where the notation $\rho = \kappa\gamma$ and κ is the preamplifier gain factor, γ denotes a piezoelectric constant [11]. Here d_b is the displacement at location b on this beam. So we put the output data in Equation (5), then it reduces to

$$[M]\{\ddot{d}\} + [K]\{d\} = \{F(a,t) - \frac{1}{\rho}(m\ddot{d}_b + c\dot{d}_b + kd_b)\} \tag{6}$$

Equation (6) can be rearranged to the following equation by using proportional damping matrix $[C] = \alpha[M] + \beta[K]$, where α and β are constants.

$$[M]'\{\ddot{d}\} + [C]'\{\dot{d}\} + [K]'\{d\} = \{F(a,t)\} \tag{7}$$

in Eq.(7), when $x = b$,

$$m_b' = m_b + \tfrac{1}{\rho}m\,,\quad c_b' = c_b + \tfrac{1}{\rho}c \quad k_b' = k_b + \tfrac{1}{\rho}k$$

where m_b', c_b' and k_b' are the matrix elements of $[M]'$, $[C]'$ and $[K]'$ on the location x=b. When the initial condition $\ddot{d}_0$, $\dot{d}_0$ and d_0 is given, the Equation (7) may conveniently be written in matrix form by using the average acceleration method [17],

$$\begin{bmatrix} K' & C' & M' \\ 0 & I & -\frac{\Delta t_i}{2} \\ I & 0 & -\frac{\Delta t_i^2}{4} \end{bmatrix} \begin{Bmatrix} d_{i+1} \\ \dot{d}_{i+1} \\ \ddot{d}_{i+1} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & \frac{\Delta t_i}{2} \\ I & \Delta t_i & \frac{\Delta t_i^2}{4} \end{bmatrix} \begin{Bmatrix} d_i \\ \dot{d}_i \\ \ddot{d}_i \end{Bmatrix} + \begin{Bmatrix} F_i \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

or

$$\Phi_1 D_{i+1} = \Phi_0 D_i + F_i \tag{9}$$

where $D_{i+1} = [\ddot{d}_{i+1} \quad \dot{d}_{i+1} \quad d_{i+1}]^T$

By transferring Φ_1 to the right side of the equation, this recursion relation can then be written as

$$D_{i+1} = AD_i + \Gamma F_i \tag{10}$$

then $\Gamma = \Phi_1^{-1},\; A = \Gamma\Phi_0$.

If the associate process noises are considered; the state equations can be transformed to the measurement equation as following.

$$D_{i+1} = AD_i + \Gamma[F_i + w_i] \tag{11}$$

$$Z_i = HD_i + v_i \tag{12}$$

where w_i is the statistical representation the input disturbance or process noises which is assumed to be zero mean and white with variance $E\{\omega_i\,\omega_j^T\} = Q\,\delta_{ij}$. D_{i+1} is the state vector, A represents the state transition matrix, Γ denotes the input matrix. Z_i represents the observation vector and H is the measurement matrix. The variance of v is the measurement noise vector. This "v" is assumed to be zero mean and white variance $E\{v_i v_j^T\} = R\,\delta_{ij}$.

2. Recursive input estimation approach

In the previous section, we derived the discrete-time state equations of the mass-damping-stiffness structures system, which is subjected to impulsive loads. However, input force estimation is a process of determining the applied loadings from an accelerometer of the system responses. The current input estimation method consists of two parts: the Kalman filter and a recursive least squares algorithm. The Kalman filter is used to generate the residual innovation sequence by observation data. Then a recursive least-squares algorithm computes the onset time history of the excitation forces by utilizing the residual innovation sequence. The detailed derivation of this scheme can be found in the content of Tuan *et al.* [5]. The association between the mathematical equations is shown in Fig. 2.

The Kalman filter equations, without external forces, are given by [18]

$$\overline{D}(i+1/i) = A \cdot \overline{D}(i/i) \tag{13}$$

$$P(i+1/i) = AP(i/i)A^T + \Gamma Q \Gamma^T \tag{14}$$

$$S(i+1) = HP(i+1/i)H^T + R \tag{15}$$

$$K_a(i+1) = P(i+1/i)H^T S^{-1}(i+1) \tag{16}$$

$$P(i+1/i+1) = [I - K_a(i+1)H]P(i+1/i) \tag{17}$$

$$\overline{Z}(i+1) = Z(i+1) - H\overline{D}(i+1/i) \tag{18}$$

$$\overline{D}(i+1/i+1) = \overline{D}(i+1/i) + K_a(i+1)\overline{Z}(i+1) \tag{19}$$

where P denotes the filter's error covariance matrix, $S(i+1)$ represents the innovation covariance, and K_a is the Kalman gain. From [5], the equations for a recursive least-squares algorithm are

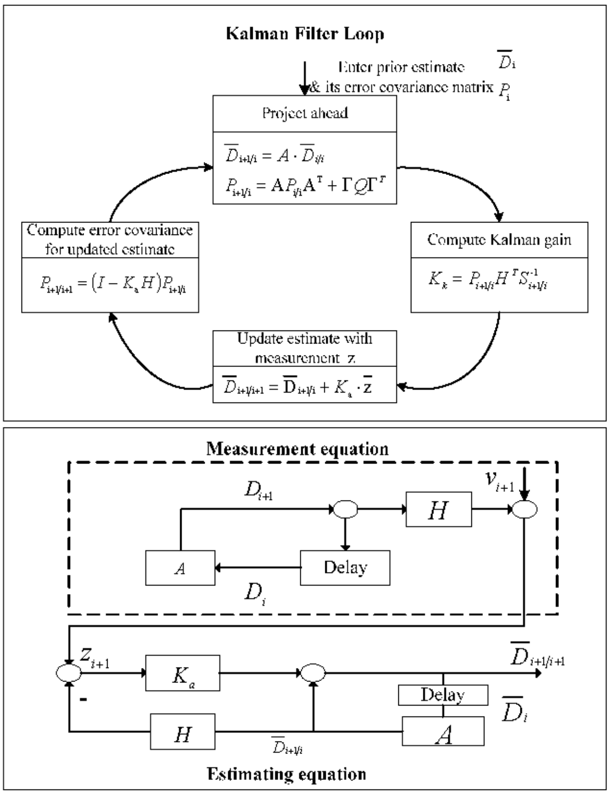


Fig. 2 Estimation flowchart

$$B_s(i+1) = H[AM_s(i) + I]\Gamma \tag{20}$$

$$M_s(i+1) = [I - K_a(i+1)H][AM_s(i) + I] \tag{21}$$

$$K_b(i+1) = \frac{1}{\gamma} \cdot \frac{P_b(i)B_s^T(i+1)}{B_s(i+1)\gamma^{-1}P_b(i)B_s^T(i+1) + S(i+1)} \tag{22}$$

$$P_b(i+1) = [I - K_b(i+1)B_s(i+1)]\gamma^{-1}P_b(i) \tag{23}$$

$$\hat{F}(i+1) = \hat{F}(i) + K_b(i+1) \cdot [\overline{Z}(i+1) - B_s(i+1)\hat{F}(i)] \tag{24}$$

where $B_s(i+1)$ and $M_s(i+1)$ are the sensitivity matrices. $\overline{Z}(i+1)$ denotes the innovation, $K_b(i+1)$ describes the correction gain for the updating $\hat{F}(i+1)$, $P_b(i+1)$ represents the error covariance of the estimated input vector, and $\hat{F}(i+1)$ is the estimated input vector. The scalar parameter γ, i.e., fading memory factor, is employed in the current algorithm to compromise between the fast adaptive capability and the loss of estimate accuracy. In this algorithm, $0 < \gamma \le 1$ is used.

The procedures are to estimate the unknown input forces by using the inverse method. They are summarized as follows.

(1). The governing equation of motion for the beam structure with accelerometer was discritized to the matrix form by using finite element formulation.

(2). The matrix form of governing equation of motion was transformed to the state-space equation by average acceleration method.

(3). We use the Kalman filter equations that are Eq. (13)-(19), to predict node displacement, and to obtain the innovation covariance S $(i+1)$, innovation $\overline{Z}(i+1)$, and Kalman gain K_a $(i+1)$.

(4). We also use the recursive least-squares algorithm that is Equation (20)-(24), to estimate the unknown external forces $\hat{F}(i+1)$.

NUMERICAL SIMULATIONS AND RESULTS

In order to demonstrate the accuracy and efficiency of the current method in estimating unknown external forces, numerical simulation of a cantilever beam, a sim-

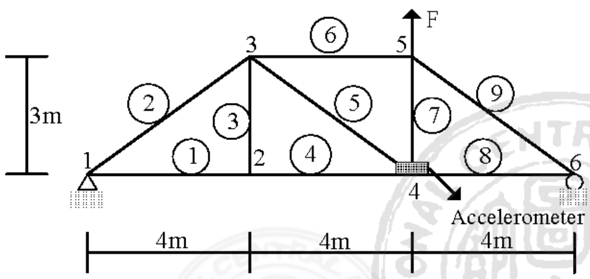


Fig. 3 Bridge truss structure

Table 1 The material specification and dimensions of beam

Beam variables	
Density (kg/m^3)	7860
Elastic modulus (GPa)	200
Length (m)	1
Cross-sectional area (m^2)	0.0004
Moment of inertia (m^4)	0.33×10^{-8}

Table 2 The material specification and dimensions of bridge truss

Truss variables	
Density (kg/m^3)	7860
Elastic modulus (GPa)	200
Cross-sectional area (m^2)	0.0005

Table 3 The material data of accelerometer

Accelerometer variables	
Mass (kg)	0.2
Natural frequency (rad/s)	24500
Damping	0.085

Table 4 Comparison of FEM and analytic functional results of the natural frequencies

Mode	FEM (rad s^{-1})	Exact solution (rad s^{-1})	Error
1	203	199.5	1.7543 %
2	1272.2	1260.2	0.9522 %

ply supported beam and a bridge truss (as Fig. 3) are investigated. The material data and dimensions of beam, truss and accelerometer are given in Table 1, Table 2 and Table 3. The initial conditions for the input estimator are generally given by $\overline{D}(0/0)=[0\,0\,0\,0\,0]^T$, $P(0/0)=\text{diag}[10^9]$, $P_b(0/0)=\text{diag}[10^8]$, and $\hat{F}(0)=[0\,0\,0\,0\,0]^T$. $M(0)$ is set to be a zero matrix for the recursive least-squares algorithm. The estimators are initialized with $P(0/0)$ and $P_b(0/0)$ as extremely large numbers, thereby causing the estimator to "ignore" the few initial estimates. In the estimation problem, the process noise covariance matrix Q and the measurement noise covariance matrix R of the input estimation algorithm are all unknown values. According to several simulation tests, this paper chooses $Q=10^{-10}$ and $R=10^{-14}$ as our simulation parameters. These selections have the effects of treating the initial errors as very large and the estimator will 'ignore' the few estimations [7].

A comparison of the FEM and analytic functional natural frequencies for free vibration of a uniform beam is shown in Table 4.

There are six types in numerical simulation examples that employed in this paper. Cantilever beam with (1) two-rectangular force, (2) two-cosine synthetic force. Simply supported beam with (3) two-rectangular force, and (4) triangular force with rectangular force were simulated. Bridge truss with (5) two-rectangular force, and (6) the exponential decay multiplies a cosine-distributed load were simulated. The simulated measurements of the beam (truss) structure are loaded into the inverse estimation algorithm, i.e., Equation (13)-(24), to identify the corresponding external forces.

After trying different initial values for the notation ρ in Eq. (6), we have found that $\rho=8\times10^7$ is a good value compared to the displacement of the forward solution. All of these simulations were done on the MATLAB software environment. From Fig. 4 to Fig. 9, part (a) shows the exact input acceleration of an accelerometer by the forward solution, part (b) depicts displacement of display from an accelerometer and exact displacement at specific location of the beam (truss) (i.e. a solid line indicates exact displacement and a dotted line indicates measurement displacement), and part (c) presents the exact and estimated external forces.

1. Estimation of the cantilever beam with two-rectangular shapes force

In the first numerical simulation example, the cantilever beam is subjected to a concentrated force that acting on the point $a=L/4$. An accelerometer was put on the location $b=L$ of this beam. The external force with respect to time domain is introduced as below

$$F(x,t)=p_0 f(t)\delta(x-a)$$

$$f(t)=0,\quad \frac{t_f}{3}\le t\le\frac{2t_f}{3}$$

$$f(t)=p_0,\quad 0\le t\le\frac{t_f}{3}\ \text{and}\ \frac{2t_f}{3}<t\le t_f$$

where $t=k\Delta t=k(t_{i+1}-t_i)$, and the final time t_f is given by 0.5 second, $\Delta t=0.0005$ second, and $p_0=20$ N.

Fig. 4 depicts the corresponding time histories of the displacement and estimation result. The result reveals a very good estimating ability.

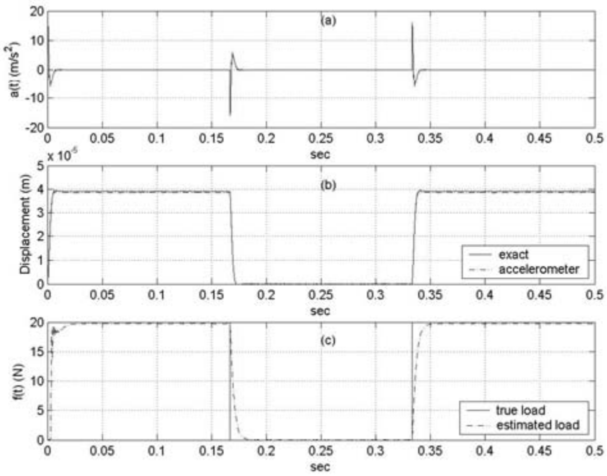


Fig. 4 Time histories of the two-rectangular shapes force

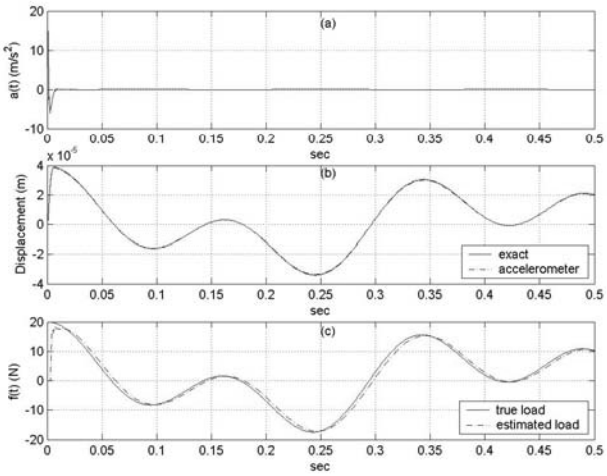


Fig. 5 Time histories of the two-cosine synthetic shapes force

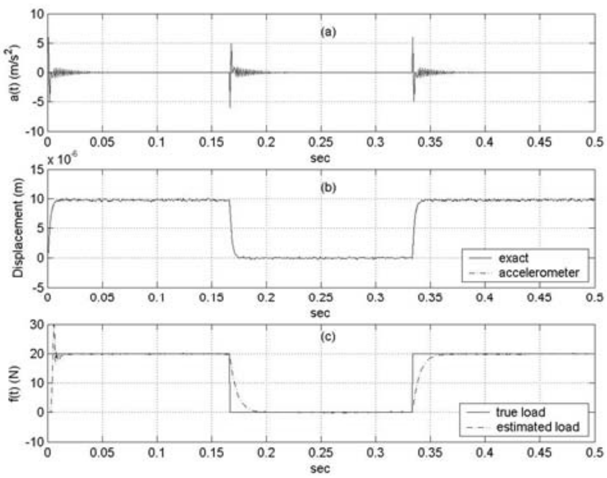


Fig. 6 Time histories of the two-rectangular shapes force

2. Estimation of the cantilever beam with two-cosine synthetic shapes force

In the third demonstration, two-cosine synthetic shapes force with respect to time domain is assumed as

$$f(t) = \frac{p_0}{2}\cos(12\pi t) + \frac{p_0}{2}\cos(5\pi t), \qquad 0 \le t \le t_f$$

The simulation parameters are the same as the previous simulation examples. Fig. 5 shows the estimation results of a two-cosine synthetic shapes force in time domain.

Next we considered the estimation of a concentrated force that acted on $L/4$ of a simply supported beam and an accelerometer that acted on $L/2$ of a simply supported beam. In following numerical simulation example, the simulation parameters are the same as previous that.

3. Estimation of a simply supported beam with two rectangular shapes force

The simply supported beam is subjected to a concentrated force that acting on the point $a = L/4$. An accelerometer was put on the location $b = L/2$ of this beam. The rectangular shapes force with respect to time domain is introduced as below

$$F(x,t) = p_0 f(t)\, \delta(x-a)$$

$$f(t) = 0, \qquad \frac{t_f}{3} \le t \le \frac{2t_f}{3}$$

$$f(t) = p_0, \qquad 0 \le t \le \frac{t_f}{3}, \text{ and } \frac{2t_f}{3} < t \le t_f$$

Fig. 6 depicts the corresponding time histories of the displacements and estimation results. Obviously the displacement of a simply supported beam is smaller than that of a cantilever beam. In Fig. 6(a) shows slight fluctuations that are due to damping matrix in Eq. (7).

4. Estimation of a simply supported beam with the triangular and rectangular shapes force

The shape of concentrated load is expressed as

$$f(t) = -\frac{3p_0}{t_f}\left(t - \frac{t_f}{3}\right), \qquad 0 \le t \le \frac{t_f}{3}$$

$$f(t) = 0, \qquad \frac{t_f}{3} \le t \le \frac{2t_f}{3}$$

$$f(t) = p_0, \qquad \frac{2t_f}{3} \le t \le t_f$$

The calculated results indicate that the displacement of a simply supported beam is same as the above section.

5. Estimation of the bridge truss with two-rectangular force

A bridge truss is subjected to a concentrated force $F(t) = p_0 f(t)$ acting on the node 5 and an accelerometer putting on the node 4. The rectangular force with respect to time domain is introduced as below

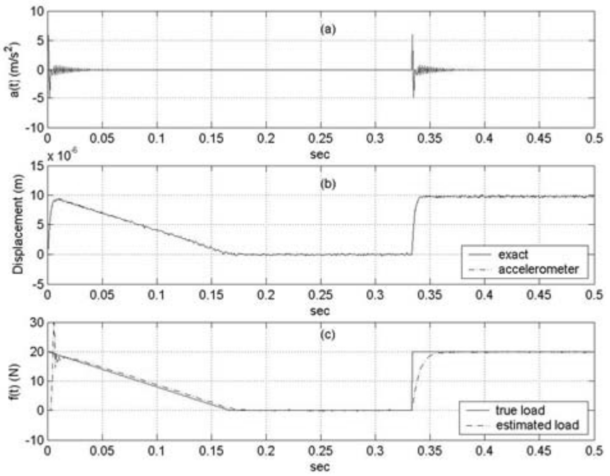


Fig. 7 Time histories of the triangular and rectangular shapes force

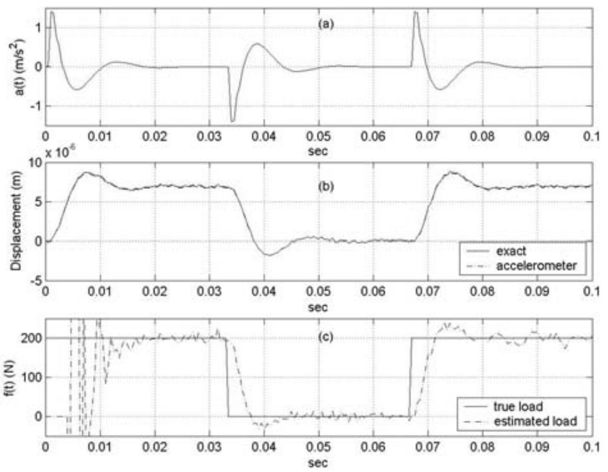


Fig. 8 The estimated input force for two-rectangular force

$$f(t) = 0, \quad \frac{t_f}{3} \le t \le \frac{2t_f}{3}$$

$$f(t) = p_0, \quad 0 \le t \le \frac{t_f}{3} \text{ and } \frac{2t_f}{3} < t \le t_f$$

where $t = k\Delta t = k(t_{i+1} - t_i)$, and the final time t_f is given by 0.1 s, $\Delta t = 0.0005$ s, and $p_0 = 200$ N.

Fig. 8 depicts the corresponding time histories of the displacement and estimation result.

6. Estimation of the bridge truss with the exponential decay multiplies a cosine-distributed load

In this numerical simulation example is still a bridge truss, the exponential decay multiplies a cosine-distributed load is expressed as

$$f(t) = p_0 \exp(-25t)\cos(20\pi t), \qquad 0 \le t \le t_f$$

The simulation parameters are the same as the previous test case. Fig. 9 depicts the time history of the displacement and estimation result of the exponential decay multiplies a cosine-distributed load.

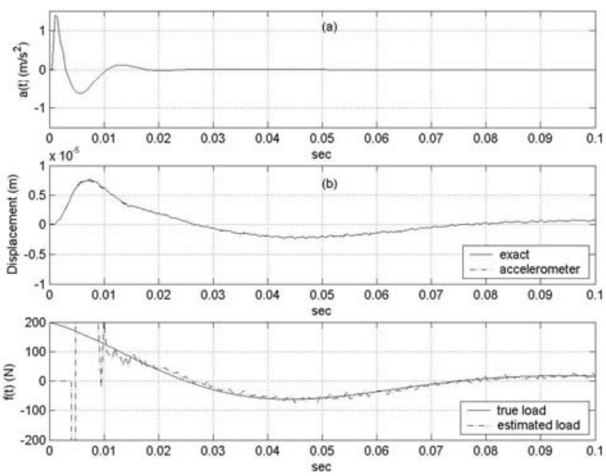


Fig. 9 The estimated input forces for the exponential decay multiplies a cosine-distributed load

DISCUSSIONS

1. There are two parts included in the Kalman filter, prediction and innovation. The Kalman filter predicts the displacement of the beam (truss) where the accelerometer located. Then Kalman filter can also innovate the displacement by using the displacement measured from accelerometer.
2. The least-square algorithm is used to reduce the error between estimated external forces and exact external forces.
3. The advantage of present inverse method is to estimate the displacement of any location on the different beam and the external forces by just put an accelerometer on any location of the different beam.
4. Obviously, the acceleration of specific point of the beam (truss) will be influenced by different kind of the external forces, and the acceleration will suddenly be altered by the impulse force as showed in part (a) of Fig. 4 to Fig. 9.
5. Part (b) of figures 4 to Fig. 7 shows the exact displacement and estimated displacement of beam. The error between these two displacements is about 1.2%, which can apply to any kind of external forces.
6. As indicated in part (c) of Figs. 4 to 9, the errors in the initial estimation are large, but after a few seconds iteration, the estimation converges to its correct values rapidly. These results show that the proposed technique can correct the error between the initial estimation values and the exact values by increasing the value of the error covariance matrix $P(0/0)$ and $P_b(0/0)$.

7. The application of the current inverse method facilitates estimating the external forces of the different structure system in time domain. The estimation results of the different external forces with respect to time domain indicate that the current estimation algorithm is capable of dealing with different beam (truss) structural system in time domain.

CONCLUSIONS

This paper presents an on-line recursive inverse method to estimate the unknown external forces of cantilever beam, simply supported beam and bridge truss from the measured acceleration of an accelerometer. The input-output behavior of accelerometer can be described as a linear time-invariant system by a difference equation. Then we use the FEM and average acceleration method to construct the system state equation of the beam structure and the accelerometer. This system state equation is then used by Kalman filter with least-squares algorithm to estimate the different distributed external forces of the beam (truss) structural system. Numerical results confirm that the proposed method can accurately estimate the different distributed external forces even the measured displacements noises were exist.

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結構系統採用逆向方法估測外力之研究

賀增原

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關鍵詞：加速計，卡爾曼濾波器、有限元素法、平均加速度法、遞迴最小平方法

摘　　要

本文模擬加速計數據在不同時域，探討結構系統位移及外力估測量。特別是處理在外力是未知的情況。結構的系統是由加速計與樑(桁架)所構成，整理系統成統御方程式。接著，運用有限元素法，將統御方程式離散成爲矩陣式，然後利用平均加速度法將矩陣式，建構成卡爾曼濾波器的狀態方程式與量測方程式。卡爾曼濾波器對於外力趨勢主要採用預測、更新與濾波的步驟。最後，採用遞迴最小平方法去降低介於量測與估測外力的誤差，以求出未知外力，同時藉由不同時域，外力作用在樑(桁架)特定的位置，進行數值模擬探討與分析，可以展現目前方法正確性。

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