應用灰色理論的競爭式學習網路於平均值/差值轉換域之向量量化技術

Vector Quantization using Grey-based Competitive Learning Network in the MDT Domain

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植基於向量量化，一個應用灰色理論的競爭式學習網路於平均值/差值轉換域技術被提出。本篇論文中，灰色理論被應用到一個兩層的修正競爭式學習網路上。其目的在於建立一編碼簿使得介於訓練向量與編碼簿中之編碼向量的灰關聯度最大。影像資訊並經由平均值/差值轉換後，以詳細係數做向量量化。根據實驗結果顯示，基於灰色理論最大關聯準則之競爭式學習網路及於平均值/差值轉換域上所產生的影像壓縮編碼簿具有效性及良好效能。

關鍵詞: 向量量化，競爭式學習網路，灰色理論，平均值/差值轉換。

Abstract

Based on Vector Quantization (VQ), a Grey-based Competitive Learning Network (GCLN) in the Mean value / Difference value Transform (MDT) domain is proposed. In this paper, the grey theory is applied to a two-layer Modify Competitive Learning Network (MCLN) in order to generate optimal solution for VQ. In accordance with the degree of similarity measures between training vectors and codevectors, the grey relational analysis is used to measure the relationship degree among them. The information transformed by mean value / difference value operation was separated into mean value and detailed coefficients. Then the detailed coefficients are trained using the proposed method to generate a better codebook in VQ. The compression performances using the proposed approach are compared with GCLN and conventional vector quantization LBG method, experimented results show that valid and promising performance can be obtained using the GCLN and proposed approach.

Key words: Vector quantization, Competitive learning network, Grey theorem, Mean value / Difference value transform.

1. Introduction

The Mean value / Difference value Transform (MDT) is the simplest wavelet transform and wavelet coding [1-2] that plays an important role in data compression. Although wavelet coding is very similar to subband coding in applications, it was originally developed from mathematical research. It provides successful application not only in compression but also in image processing and so on.

Grey system theory was proposed in 1982 [3-4] and has been successfully implemented on many fields such as image compression [5-6], high noise vehicle plate recognition [7], and earthquake analysis [8] etc.

A number of vector quantization algorithms for image compression have been proposed over the years [9-11]. The purpose of vector quantization is to create a codebook such that the average distortion between training vectors and their corresponding codevectors in codebook is minimized. Codebook design can be considered as a clustering process in which each training vector is classified into a specific class. The clustering process updates the codebook iteratively such that the average distortion between training vectors and codevectors in the codebook becomes smaller and smaller.

Neural networks with competitive learning have been demonstrated capable of performing vector quantization [12-15]. In addition to the neural network-based techniques, the grey relational theory has also been demonstrated to address codebook design in this paper. In GCLN, the learning rule and stopping criterion of the original competitive learning neural network are modified, and the grey relational strategy is added to address codebook design problems. The problem of the VQ is regarded as a process of the minimization of a object function. This object function is defined as the average distortion between the training vectors in a divided image to the cluster canters represented by the codevectors in the codebook. The modify competitive learning network is simpler than that of the conventional competitive leaning network, and is constructed as a two-layer fully interconnected array with the input neurons representing the training vectors and output neurons representing the codevectors in the codebook.

In the mean value / difference value transform domain, this object function is defined as the average distortion between the training vectors in detailed information to the cluster centers represented by the codevectors in the codebook. The training vectors, constructed by detailed information, are directly fed into this GCLN. In a simulated study, the GCLN is demonstrated to have the capability for VQ, and in the mean value / difference value transform domain, the proposed approach also is shown the promising results in image compression.

2. Competitive Learning Network

A competitive learning network is an unsupervised network used to select a winner based on similarity measure over the feature space. Especially, a proper neuron state is updated if and only if it wins the competition among all neurons. Many schemes for competitive learning networks have been proposed [16-17].

In the simple competitive learning networks, the single output layer consists of cluster centers, each of which is fully connected to the inputs via interconnection strength. In conventional competitive learning only one output unit is active at a time and the objective function is given by

$$J_c = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}\left\|\mathbf{x}_i - \boldsymbol{\omega}_j\right\|^2 \tag{1}$$

where n and c are the number of training vectors and the number of clusters respectively. $u_{i,j} = 1$ if $\mathbf{x}_i$ belongs to cluster j and $u_{i,j} = 0$ for all other clusters. The neuron that wins the competition is called the winner-take-all neuron. Thus $u_{i,j}$ indicates whether the input sample $\mathbf{x}_i$ activates neuron j to be a winner. $u_{i,j}$ is given by $u_{i,j} = \begin{cases} 1 & if \left\|\mathbf{x}_i - \boldsymbol{\omega}_j\right\| \le \left\|\mathbf{x}_i - \boldsymbol{\omega}_k\right\|, \quad for \quad all \quad k \\ 0 & otherwise. \end{cases}$; (2)

The incremental $\Delta\boldsymbol{\omega}_j$ is given by [16-17]

$$\left\langle \Delta\boldsymbol{\omega}_j \right\rangle = -\eta \frac{\partial J_c}{\partial \boldsymbol{\omega}_j} = \eta \sum_{i=1}^{n} \left(\mathbf{x}_i - \boldsymbol{\omega}_j\right) u_{i,j}, \quad j = 1,2,\Lambda ,c. \tag{3.a}$$

where η is the learning-rate parameter. Although Eq. (3.a) is written as a sum over all samples, practically it is usually used incrementally, i.e.

$$\Delta\boldsymbol{\omega}_j = \eta\left(\mathbf{x}_i - \boldsymbol{\omega}_j\right) u_{i,j}, \quad j = 1,2,\Lambda ,c. \tag{3.b}$$

The updating rule is given by

$$\boldsymbol{\omega}_j(t+1) = \boldsymbol{\omega}_j(t) + \Delta\boldsymbol{\omega}_j(t). \tag{4}$$

The Modified Competitive Learning Network (MCLN) has the same architecture as the conventional competitive learning network [15]. It is an unsupervised competitive learning network using the modified competitive learning rule and stopping criterion. Similarly to the standard competitive learning rule in Eqs. (3.b) and (4), the least squared error solution can be obtained by [15]

$$\boldsymbol{\omega}_j(t+1) = \begin{cases} \boldsymbol{\omega}_j(t) + \eta(\mathbf{x}_i - \boldsymbol{\omega}_j) & if \left\|\mathbf{x}_i - \boldsymbol{\omega}_j\right\| \le \left\|\mathbf{x}_i - \boldsymbol{\omega}_k\right\|, \quad for \quad all \quad k \quad ; \\ \boldsymbol{\omega}_j(t) & otherwise. \end{cases} \tag{5}$$

The MCLN algorithm modifies only output neurons without updating the interconnection strengths . Instead of updating the interconnection strengths using the winner-take-all scheme in the conventional competitive learning network and for the purpose of simplifying the hardware architecture, the MCLN only modifies the output states (cluster centroids).

3. Grey-based Competitive Learning Network for VQ

Suppose an image is divided into n blocks (vectors of pixels) and each block occupies $\lambda\times\lambda$ pixels. A vector quantizer is a technique that maps the Euclidean $\lambda\times\lambda$-dimensional space $\mathbf{R}^{\lambda\times\lambda}$ into a set $\{\boldsymbol{\omega}_j, j=1,2,\ldots,c\}$ of points in $\mathbf{R}^{\lambda\times\lambda}$, called a codebook. It looks for a codebook such that each training vector is approximated as close as possible by one of the code vectors in the codebook. A codebook is optimal if the average distortion is at the minimum value. The average distortion $E[d(\mathbf{x}_i,\boldsymbol{\omega}_j)]$ between an input sequence of training vectors $\{\mathbf{x}_i, i=1,2,\ldots,n\}$ and its corresponding output sequence of code vectors $\{\boldsymbol{\omega}_j, j=1,2,\ldots,c\}$ is defined as

$$D = E[d(\mathbf{x}_i,\boldsymbol{\omega}_j)] = \frac{1}{n}\sum_{i=1}^{n} d(\mathbf{x}_i,\boldsymbol{\omega}_j) \tag{6}$$

Grey system is usually divided into several topics such as grey theory, grey mathematics, grey prediction, grey generating space, grey decision, and grey relational analysis. Grey relational theory demonstrates the measurement of similarity between training vectors and codevectors based on grey relational space. Let $\mathbf{x}_i$ be training vector and $\boldsymbol{\omega}_j$ be the codevector *j*, then the grey relational coefficient is defined as

$$\gamma\left(\mathbf{x}_i,\boldsymbol{\omega}_j\right) = \frac{\Delta_{\min.} + \xi\Delta_{\max.}}{\Delta_{ij} + \xi\Delta_{\max.}} \tag{7}$$

where

$$\Delta_{\min.} = \min\left|\mathbf{x}_i - \boldsymbol{\omega}_j\right|$$

$$\Delta_{\max.} = \max\left|\mathbf{x}_i - \boldsymbol{\omega}_j\right|$$

$$\Delta_{ij} = \left|\mathbf{x}_i - \boldsymbol{\omega}_j\right|$$

and $0 < \xi < 1$ is the distinguished coefficient. Then the grey relational grade is

$$\gamma_{i,j} = \frac{1}{\lambda\times\lambda}\sum_{m=1}^{\lambda\times\lambda}\gamma\left(\mathbf{x}_i,\boldsymbol{\omega}_j\right) \tag{8}$$

where *m* is the dimension of the training vector $\mathbf{x}_i$ and the codevector $\boldsymbol{\omega}_j$.

In this paper, the grey theory is applied to a two-layer MCLN in order to generate optimal solution for VQ. Then the modified competitive learning rule is modified as

$$\omega_j(t+1) = \begin{cases} \omega_j(t) + \eta(\mathbf{x}_i - \omega_j) & if \quad \gamma_{i,j} \geq \gamma_{i,k}, \quad for \quad all \quad k \quad ; \\ \omega_j(t) & otherwise. \end{cases} \tag{9}$$

Then the grey-based competitive learning network can be used for VQ in image compression summarized as follows.

Step 1: Initialize the codevectors $\omega_j(2 \leq j \leq c)$, learning rate η, maximum error (*ME*), total error (*TE*), and a threshold value ε.

Step 2: Input a training vector $\mathbf{x}_i$ and find the winner's codevector based on the maximum grey relational grade.

Step 3: Apply Eq. (9) to update the winner's codevector and set *TE*=*TE*+ Δij.

Step 4: Repeat Step 2 and 3 for all input samples, then if $(ME - TE)/ME < \varepsilon$, go to step 5; otherwise replace *ME* content from *TE*, and go to Step 2.

Step 5: Complete the codebook design.

4. Mean Value / Difference Value Transform Domain and GCLN

To introducte mean value / difference value transform, let us take an input sequence, {78, 52, 16, 28, 48, 56, 46, 30} as an example. These eight sampling values may be taken from some row of a gray value image. As follows, we take mean value / difference value operation on these eight values, after three times for transforing, the final coefficients are 44.25, -0.75, 21.5, 7, 13, -6, -4, and 8.

78	52	16	28	48	56	46	30
65	**22**	**52**	**38**	*13*	*-6*	*-4*	*8*
43.5	**45**	*21.5*	*7*	13	-6	-4	8
44.25	*-0.75*	21.5	7	13	-6	-4	8

The eight sampling values in the first row are treated as four pairs. In the second row, each bold value is the mean value of a pair in the first row: 65 = (78 + 52) / 2, 22 = (16+ 28) / 2, 52 = (48 + 56) / 2, and 38 = (46 + 30) / 2. In the third row, each bold value is also the mean value of the first two pairs in the second row. Analogously, in the fourth row, the bold value is the mean value of the first pair in the third row.

In the second row, each italic value is the difference value of a pair in the first row: 13 = (78 - 52) / 2, -6 = (16 - 28) / 2, -4 = (48 - 56) / 2, 8 = (46 - 30) / 2; we call these values by the name of detailed coefficients. Analogously, in the third row, the italic values are the difference values of the first two pairs in the second row. In the fourth row, the italic value is the difference value of the first pair in the third row.

We can be sure that the mean value / differential value transform is reversible and the eight original sampling values can be reconstructed step by step using the one mean value and seven detailed coefficients.

For a 2-D image, the input image will be decomposed into nonoverlapping blocks of equal size at the first stage. After the image blocked, the mean value / difference value transform is applied to transform each block into mean value and detailed coefficients. Then the detailed coefficients are fed into the presented GCLN algorithm for training vectors and generating better codebook. In the decoder, indexes of codevectors are received to get detailed information (codevectors). Then, the inverse mean value / difference value transform is used to reconstruct the image using mean value and detailed informations.

5. Experiment Results

The codebook design is the primary problem in image compression based on vector quantization. In this paper, the quality of the images reconstructed from the designed codebooks was compared with that from the LBG、GCLN and mean value / difference value transform + GCLN (named by MDT+GCLN) methods. The training vectors were extracted from 256×256 with 8-bit gray level real images, each of which is divided into 4×4 blocks to generate 4096 non-overlapping 16-D training vectors. Three codebooks of size 64, 128, and 256 were built by these training vectors. The resulting images were evaluated subjectively by the mean squared error (MSE) and peak signal to noise ratio (PSNR) that is defined for images of size N×N as

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \tag{10}$$

and

$$MSE = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_{ij} - \hat{x}_{ij}\right)^2 \tag{11}$$

where x_{ij} and $\hat{x}_{ij}$ are the pixel gray levels from the original and reconstructed images, and 255 is the peak gray level. Table 1 shows the PSNR and MSE of the "F16", "Girl" and "Lena" images reconstructed from three codebooks of size c=64, 128, and 256 designed by the LBG and the presented GCLN and MDT+GCLN methods. From experimental results, the reconstructed images obtained from the presented GCLN are superior to those obtained from the LBG algorithm, and the performance of the presented MDT+GCLN algorithm is significantly better than the GCLN algorithm.

6. Conclusions

In this paper, a two-layer modify competitive learning neural network based on gray relational theory for VQ in the mean value / difference value transform domain has been presented. Instead of updating the interconnection strengths using the winner-take-all scheme in the conventional competitive learning network, the GCLN algorithm only modifies output neurons and omits the updating of the interconnection strengths. The information transformed by mean value / difference value operation was separated into mean value and detail coefficients. Then the detailed coefficients are trained using the presented method to generate a better codebook in VQ. From the experiment results, the presented GCLN and MDT+GCLN methods produce reconstructed images more promising than those reconstructed by the LBG method.

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Table 1. PSNR and MSE of the images reconstructed from codebooks of various size designed by the LBG and the proposed GCLN and MDT+GCLN algorithms.

Images/Algorithms \ Codebook Sizes		64		128		256	
		PSNR	MSE	PSNR	MSE	PSNR	MSE
F16	LBG	24.11	222.31	25.29	192.30	26.34	132.94
	GCLN	24.75	217.68	25.35	189.69	26.82	135.27
	MDT+GCLN	27.04	128.65	28.07	101.48	29.58	71.63
Girl	LBG	27.68	109.62	28.51	91.60	29.69	77.09
	GCLN	28.79	85.98	29.91	66.44	31.05	51.02
	MDT+GCLN	31.83	42.70	32.70	34.92	33.51	28.99
Lena	LBG	25.26	146.89	26.37	127.01	27.06	106.71
	GCLN	26.09	160.01	27.23	123.11	28.84	84.89
	MDT+GCLN	28.88	84.25	30.18	62.46	31.03	51.34