An Enhanced Rothermel's Model for 2D Fire Spread Simulation

Xin En Luo[*] Mei-Hsia Chiang[***] Ching-Chung Chen[**]

Wen-Hong Wu[***]

ABSTRACT

Fire hazard and its destruction become a big issue around the world especially in the context of global warming. A reliable fire simulation model will be a useful tool for fire safety managers to control the destruction area. There are two major simulation methods which are raster-based method and vector-based method to simulate the fire spread in computer. Both methods have their own advantage and challenging. One of the most challenging for vector - based method is crossover problem. Regardless of which method is chosen, it is definitely a tedious, time and resource consuming process to calculate crossover until level set methods is introduced by Osher and Sethian [1] at 1988. Level set methods are well known for its ability to easily handle topological changes such as merging or breaking interfaces. Therefore the level set method can be applied to resolve the crossover issue in fire spread model using vector-based method. Level set methods need a suitable fire model to expand the surface. The first and most recent model to apply level set methods to expand fire spread is introduced by V. Mallet, D. Keyes, and F. Fendell [2]. The maturity of the model which focuses most on wind factor still requires improvement to fit the real world fire spread. Rothermel's fire model is the most commonly used fire model to simulate fire spread in America. The goal of this research is to enhance the existing Rothermel's fire model and apply level set methods to this modified model to forecast the behavior of ground fire spread in the much more complex real world simulation.

Key words: Rothermel's fire model, level set method, vector-based method

[*] Associate Professor, Department of Information Management, Hsing Wu University
[**] Associate Professor, Department of Information Management, Hsing Wu University
[**] Assistant Professor, Department of Information Management, Hsing Wu University
[***] Assistant Professor, Department of Finance, Hsing Wu University

I. Introduction

There are several kinds of fire models to simulate fire spread. The most two methods are raster-based methods (cellular discrete techniques) and vector-based method (polygon expanding techniques). Cellular techniques are to simulate fire growth as a discrete process of ignitions across a regularly spaced landscape grid referring to as cellular. A cell will be ignited by its burned neighbor cells from a certain conditional level. The polygon techniques are defined by a series of two dimensional vertices which points with xy-coordinates. The expansion of a fire polygon is determined by computing the spread rate and direction from each vertex and multiplying by the time step. For polygon techniques, crossovers problem is the most difficult issue. Crossover is two fire fronts overlap. Regardless of the methods chosen, crossover removal is expensive in time and computing power.

Level set methods are a numerical tool introduced by Osher and Sethian [1] and popular for tracking, simulating the dynamics of moving surface. It is well known for its ability to easily handle topological changes such as merging or breaking interfaces. So, it is easy to resolve the crossover issue in fire spread problem using polygon techniques. The first and most recent model to expand fire spread using level set method is introduced by V. Mallet, D. Keyes, and F. Fendell [2]. The maturity of the model which focuses most on wind factor still requires improvement to fit the real world fire spread. V. Mallet, D. Keyes, and F. Fendell use F.E.Fendell and M. Wolff 's [4] fire model to level set methods to expand the surface of fire front and have a good fire spread shape under different wind speed.

Rothermel's fire model [3] which is the most popular fire model used in the United States affords a good fire forward formula for the fire front. We will use this formula to calculate the fire expanding speed and apply the Huygen's principle and ellipse assumption to calculate the direction and distance of the expanding fire. The idea in this research model to improve the Mallets model is based on the level set methods dissecting Rothermel's fire model into multiple independent objects such as wind, terrain, and fuel. It separates the independent factors (terrain, vegetation, and wind) out from traditional fire spread formula and keeps as simple as possible for calculating the spread speed by level set methods and then combines those results together in the final to estimate the next fire spread points.

The goal of this research is to enhance the existing Rothermel's fire model and use level set methods as a expanding method to forecast the behavior of ground fire spread in the much more complex real world simulation.

This paper is organized as the following: Section II describing the existing Models of fire spread which we will use; Section III presenting the framework of our research method; Section IV giving some experiment results to demonstrate the models and section V summary.

II. Models of Fire Spread

Fire spread of fire propagation is determined by the rate of heat which is transferred from the burning fuel to unburned fuel. Computerized models of fire spread have been researched for decades. The most common research fields are cellular discrete techniques which based on raster method and polygon expanding techniques which based on vector method. This research focuses on the polygon expanding technique to resolve the crossover problem.

The fire spread rate of all fire models focuses only on the fire heading direction on a straight line. Therefore, using a polygon expanding technique to expanding fire front on the 2-D plane by a fire model is reliance on two assumptions. First, the fire growth uses Huygens principle which the fire front is propagated at the fire front edge. Second, the fire shape is an ellipse. From those two assumptions, we can simulation fire spread on 2-D or 3-D space from a one dimensional fire model.

Rothermel [3] is a famous semi-empirical fire model which is a combination of a theoretical and empirical model. It is composed of simple, general physical theories, and completed through experimentation. This fire model was published in 1972 and is the first model to simulate wild fires. It was developed from Frandsen's theory [5] which used the conservation of energy ahead of an advancing fire and assumed that the fire occurred under a homogeneous fuel bed. This model focuses on the energy transfer between neighboring fuel cells as they ignite assuming uniform fuel at the surface which burns at a height of less than two meters from the ground.

The rate of spread (ROS) in Rothermel's model is the ratio of the propagating heat flux to the energy required to dry and preheat unburned fuels until they ignite. The framework defines the heat source as a reaction intensity I_R, which is the expression of fuel load, fuel particle size, fuel chemistry, fuel arrangement, and fuel moisture. Propagating flux combines the effect of forward radiation, convection (including flame contact), and piloted ignition. It is the product of I_R and the propagating flux ratio, ξ, the latter term represents the proportion of reaction intensity that is transferred to the unburned fuels.

Rothermel's model is that the spread rate is based on an energy balance ratio of power source and heat sink terms:

$$R = \frac{\text{Power density of propagating flus}}{\text{Preheating energy of fuel}} = \frac{I_p}{\rho_b Q_T} \quad (1)$$

where I_p is the fire propagating intensity provided by the heat source, Q_T is the heat sink energy required to heat, dry, and pyrolysis the fuel, and ρ_b is the fuel bed bulk density.

As a mathematical expression, in Rothermel's model, the wildfire spread rate is given by:

$$R = \frac{I_R \xi (1 + \Phi_w + \Phi_s)}{\rho_b \, \varepsilon Q_{ig}} \quad (2)$$

- R : head fire spread rate (m/min)
- I_R : reaction intensity (kJ/min • m^2)
- ξ : the propagating flux ratio
- ρ_b : over-dry bulk density (kg/m^3)
- ε : effective heating number
- Q_{ig} : Heat of ignition
- Φ_w : wind coefficient
- Φ_s : slope factor

Reaction intensity I_R:
The reaction intensity I_R is the rate at which burning grasses release energy. The unit kj is 'kilojoule'.

The propagating flux ratio ξ:
This is the basic flux component with no wind (Φ_w) or slope (Φ_s). The Rothermel model assumes that the reaction intensity I_R is independent and can be correlated with the propagating flux such that $I_R\xi = f(I_R)$.

Over-dry bulk density ρ_b :
This fuel density combines with other parameters to give $\rho_b = w_o/\delta$ where w_o is the over dry fuel loading and δ is the fuel depth.

Effective Heating Number ε:
ε is the ratio of the effective bulk density $\rho_{b\varepsilon}$ to the actual bulk density ρ_b. ε is always less than one and approaches zero as the fuel size increases.

Heat of ignition Q_{ig}:

Q_{ig} describes the heat required to ignite fuel. It depends on:

- ignition temperature
- the ratio of fuel moisture
- fuel size, that is the amount of fuel available

Wind Φ_W:

$$\Phi_w = C(3.28U)^B \left(\frac{\beta}{\beta_{op}} \right)^{-E} \tag{3}$$

where C, B and E are functions of the fuel particle size in the fuel bed and β is the packing ratio of the fuel bed.

Terrain Φ_s:

$$\Phi_s = 5.275 \beta^{-0.3} (\tan \vartheta_t)^2 \tag{4}$$

where ϑ_t is the slope and β is the packing ratio of the fuel bed.

Rothermel's model computes the steady-state fire spread rate in one-dimension parallel to the ground surface. Since Rothermel's model only predicts a one-dimensional fire spread in the heading direction, the assumption of an elliptic fire shape is crucial. Based on this assumption, we are able to calculate the frank and back fire with wind on the vector-based method.

III. Strategy and Resolution

The vector-based method, using polygon expanding techniques, is one of the major fire simulation methods in computerized models where the fire front is defined by two-dimensional vertices on the xy- coordinates. The fire growth depends on the spread rate and direction of each vertex. The number of vertices increases as fire expands over time. The vector based method is based on two assumptions. The first assumption is Huygens' principle which states that each vertex on the fire-front is independent of the others and expands in the same way from the previous point. Based on this assumption, the fire growth is calculated on the fire front and is propagated at the fire front edge to an unburned area. This assumption requires information such as time, direction, and the rate of fire spread for the points on the fire front edge. The second assumption is that the fire shape is an ellipse under the wind. With these two assumptions, we can now successfully apply the one-dimensional fire spread model to the fire front points to get the rate of fire spread on the fire head and calculate the direction for the fire expansion to form the fire shape. Mallet et al. [2] modified Fendell and Wolff's fire model [4] to Eq. (4) and used them in level set methods to simulate a simple fire. Mallet 's fire model is

$$\begin{aligned} F(U,\vartheta) &= \varepsilon_0 + a\sqrt{U\cos^n\vartheta}, && \text{if } |\vartheta| \le \pi/2 \\ F(U,\vartheta) &= \varepsilon_0(\alpha + (1-\alpha)|\sin\vartheta|), && \text{if } |\vartheta| > \pi/2 \end{aligned} \tag{5}$$

where ε_0, a are parameters depending on fuel and $\alpha \in [0,1]$ is ratio between the velocity at the rear and flanks. Based on field tests, empirical and plausibility, Mallet et al. [2] suggested to set $n = 3$ to have a significant fire shape from the flanks to the head.

Mallet et al. applied the level set method to a simplified fire model Eq. (5), and obtained an elliptical fire shape. Nevertheless, this model can be further studied and modified to a more complex environment such as under different kinds of weather, terrain, fuel bed, etc. The model Mallet et al. [2] used has only two conditions including fuel and wind factors. They are part of the same series of factors of Rothermel's model. We kept all the factors of the Rothermel's model except replacing the wind factor with Mallet's model to produce two dimensional simulation under the frank and back wind direction.

We briefly revisited Rothermel's Model Eq. (2). This model has two characteristics. First, the fire spread rate focuses on the fire heading direction only. Second, this model can be decomposed into three independent parts. For the 'head fire direction only' characteristic, it is necessary to assume that the fire spread shape is elliptical. Otherwise we cannot estimate the fire spread direction except for the head fire. Under this assumption, Mallets's parameters provide a good solution if we consider the wind factor only. We only need the fire speed for head wind and use Mallets's parameters to calculate and predict the speed of the expanding fire in the other directions.

From Rothermel's model Eq. (1), if we write the fuel factor $\frac{I_R\varepsilon}{\rho_b \varepsilon Q_{ig}}$ to a function f and denote $E_0 = f(I_R, \xi, \rho_b, \varepsilon, Q_{ig})$, then Rothermel's fire model can be simplified to

$$R = E_0(1 + \Phi_w + \Phi_s) \tag{6}$$

where Φ_w is factor of wind in Eq. (3) and Φ_s is factor of terrain in Eq. (4). Similarly, we simplify the wind factor, and write $A = g(C, \beta, \beta_{op}, E)$ which $g = C\left(\frac{\beta}{\beta_{op}}\right)^{-E}$ and let $U' = 3.28U$. Then, Eq. (6) can be rewritten to

$$R = E_0(1 + A\,U'^B + \Phi_s)$$

If a fire is on the zero elevation with $\vartheta = 0$, then $\Phi_s = 5.275\,\beta^{-0.3}(\tan\vartheta\,)^2 = 0$. Eq. (6) becomes

$$R = E_0(1 + A\,U'^B) \tag{7}$$

This fire spread depends on fuel E_0 and wind $A\,U'^B$ only.

We next transform Mallet's model Eq(5) to a new form

$$\begin{aligned} F(U,\vartheta) &= \varepsilon_0 + aU^{1/2}(\cos\vartheta)^{3/2}, && \text{if } |\vartheta| \le \pi/2 \\ F(U,\vartheta) &= \varepsilon_0(\alpha + (1-\alpha)|\sin\vartheta|), && \text{if } |\vartheta| > \pi/2 \end{aligned} \tag{8}$$

Although it has no direct impact, following Mallet's suggestion, we replaced n with 3 here and focused on the headwind ($\vartheta = 0$) only. Mallet's model Eq. (8) can be rewritten as

$$\begin{aligned} F_H(U,\vartheta) &= \varepsilon_0 + aU^{1/2}, && \text{if } |\vartheta| \le \pi/2 \\ F(U,\vartheta) &= \varepsilon_0, && \text{if } |\vartheta| > \pi/2 \end{aligned} \tag{9}$$

F_H is the head wind direction. For any case, Eq. (7) and Eq. (9) must generate the same results because the equations consist of the same element of fuel and wind only to calculate the fire spread. Furthermore, if we consider fuel factor only for the fire-spread simulation, we can assume $E_0 = \varepsilon_0$ to generate the same results. The reason to have the same results from the above two equations is that the equations consist of the same element of fuel. Therefore, we can define a new model as

$$\begin{aligned} R &= \varepsilon_0 + a\sqrt{U\cos^3\vartheta} + \varepsilon_0\Phi_s, && \text{if } |\vartheta| \le \pi/2 \\ R &= \varepsilon_0(\alpha + (1-\alpha)|\sin\vartheta|) + \varepsilon_0\Phi_s, && \text{if } |\vartheta| > \pi/2 \end{aligned} \tag{10}$$

and

$$\Phi_s = 5.275\beta^{-0.3}(\tan\vartheta)^2$$

Traditionally, people use the letter F to represent the velocity vector on the norm direction in the level set method and use the letter W to represent the wind factor in the fire model. We will follow this tradition to use the letter F replacing the letter R; use $W(U, \vartheta)$ replacing the wind factor $F(U, \vartheta)$ in the previous equation and apply to level set methods.

IV. Experiment Result and Analysis

From above section, we already have a fire model and still need a expanding method to computerize the fire spread in computer. We will use level set methods as our expanding method. In the level set method, a moving surface can be described by a signed distance function $\varphi(x)$ which returns the distance from a given point in space to the surface. Points inside the surface have a negative distance, and those outside have a positive distance from the surface. So, the set of points with distance zero, the zero level set $\varphi(x, t) = 0$, represents the moving surface at time t. The surface evolution is modeled using partial differential equations (PDE). Osher and Sethian [1] give the level set equation as

$$\varphi' = -V \cdot \nabla\varphi \tag{11}$$

where φ is zero level set, $V=x'(t)$ represented the velocity field, and $\nabla\varphi$ is the gradient of φ with respect to x at time t. For fire front propagation, based on Huygens' principle, the vector field moves forward in the normal direction of the surface. Assuming at a given point $x \in R^n$ and the speed F at x, the velocity field V used in level set equation Eq. (11) will then be defined as

$$F = V \cdot n$$

$$\text{where } n = \frac{\nabla\varphi}{|\nabla\varphi|}$$

Replace it in Eq. (11), and we have

$$\varphi'(t) = -F \cdot |\nabla\varphi|. \tag{12}$$

F in Eq. (12) is the force on the norm direction of an expending surface and is the Rothermel's R in Eq. (10). Now we can apply Eq.(10) to Eq.(12) to get $\varphi'(t)$. Then use Euler formula Eq.(13) to find the next fire expanding point during the time t. Euler formula is

$$\varphi_{\text{new point}} = \varphi_{\text{old point}} + \Delta t \cdot \varphi'(t). \tag{13}$$

Our approach proceeds in five steps:

In the first step we scale the actual area of simulation into the computational domain and discretize the domain with a uniform mesh grid size. This is the area where we want to simulate the fire propagation.

In the next step, we save the parameters describing the fuel content (for ε_0) and elevation (for Φ_s) at that particular location for each cell to form a same size of matrix as of the domain at the last step.

In the third step we initialize the value of a distance function $\varphi(x, t)$ at the mesh. This is the starting point of the fire. The function φ is defined as φ $(x, t) =$ d where d is the distance from the point (x, y) to the interface at time $t = 0$. The positive sign is used for points (x, y) outside of the closed curve which represent unburned area; the negative sign is used inside of the closed curve representing burned area.

In the fourth step we use Eq. (10) as a vector field F for points (x, y) advecting in the direction decided by elliptical spread assumption. Calculate the F from the matrix at step two and wind parameter.

Finally we use the level set method discussed above to get the new position for φ at time t_{new}.

The pseudo-code is shown in table 2.

We present some experimental results from our new model under different factors. The parameters used for the experiments are at Table 1 in this paper. The simplest fire spread model which is on a level terrain with a homogeneous fuel and no wind shown in the Figure 1. Then simulate the fire with a west moving wind in Figure 2. From this figure, it shows the same results from our model as Mallet's level set model [2]. Then a different scenario in Figure 3 is simulated to determine the influence of the wind on the rate and pattern of the fire spread with heterogeneous fuel. The landscape consists of a pond on the west side and a small hill on the east side. This simulation is at domain [-2, 2]X[-2, 2] with resolution dx = 0.1; the other parameters remains same. Fire is ignited at (0, 0) and radius is 0.2, with west wind originating. At the beginning, the fire spreads toward the east with the wind and subsequently moves counterclockwise with changing wind directions. In the experiments, we test a simple uphill on a level ground and a simple hollow on a level ground with west wind at the beginning. Both the result show a very perfect fire spread simulation.

V. Conclusions and Future Work

The fire spread rate of Rothermel focuses on the fire heading direction only on a straight line. Therefore, we need to rely on an assumed elliptical fire shape to calculate the other directions of spread rate. We also dissect Rothermel's fire model to three independent parts - fire spread, wind and slope. Then add those extra attribute through combining with level set methods to improve the existing Matllet's model. We believe the researched model will perform better under the complex environment to meet the challenge to predict the real world fire spread.

Our research advances Matllet's model to a heterogeneous environment by modified Rothermel's fire model. The fire simulation model is verified by three fire simulations and meets the expected results. During the experiment cases, we found some parts could to be improved in the future work. First is a real world data to replicate the simulation. Second is no wind affect on the rear direction. Third is how to improve the stability (converge) of level set function when a large difference of altitude on the terrain.

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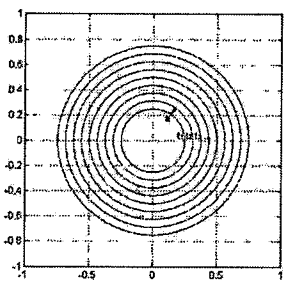


Figure 1 A basic fire spread model from the modified Rothermel's fire model by level set method.

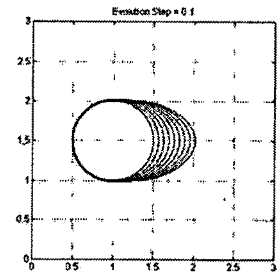
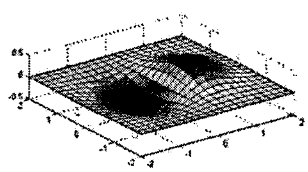
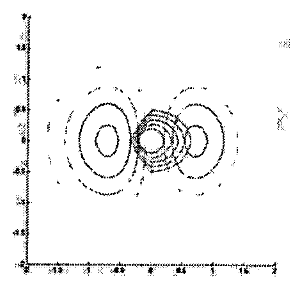


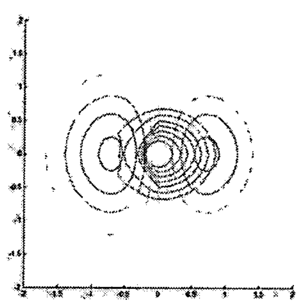
Figure 2 A perfect elliptical fire shape under a west wind.



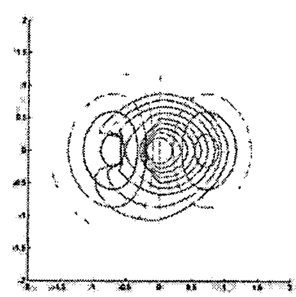
(a) A terrain with a pond on the west side (in blue) and a hill on the right side (in red).



(b) Fire go fast to east hill and very slow on the down hill direction.



(c) East wind to help the fire go west fast to pass the pond.



(d) North-east wind.

Figure 3 A fire simulated in a natural environment - A fire spreads on a terrain with a hill and a pond. Fire passes around pond with wind from different directions.

Table 1 Default values used for different simulation scenarios in our experiment cases

Parameter	Value	Parameter	Value
Domain	[-1,1]X[-1,1]	n	3
Resolution	$\Delta x = \Delta y = 0.01$	U	100
Spatial discretization	201	a	0.5
Initial front	Circle	ε_0	1.0
Circle center	(0,0)	α	0.5
Initial radius	0.25	Time step	$\Delta t = 0.1$

Table 2 Fire spread simulation process pseudo-code

```
Main
{
    Initial Data and Parameters
    {
        Get grid: min(x, y), max(x, y) and resolution dx, dy
        Get time: startTime, endTime, and timeStep
        Get Level set function data:
            if self-defined: Input φ-file
            else: Get center, radius, and shapeType
    Initialize level set equation: φ= (grid, level set function data)
    % ignition area
    Input the environment data files
    % The real time wind, topography, fuels, ...information
        Set environment parameters: envPs
        % envPs is a structure variable which values
            from environment data files
    }
    For each timestep(startTime, endTime, timeStep)
    {
        Calculate new fire front by Level Set Evolution     % below
        Graphic
        Get new envPs
    }
```

```
}

Level Set Evolution(grid, envP, φ)
{
    Get current fire front φ
    Calculate force F = (envPs)
    For each (x, y)
    {
        Calculate φ'
        Calculate |∇φ |
        Calculate CFL Time Constraint     % below
    }
    Return φnew
    }
CFL Time Constraint(φ', |∇φ |)
{
Calculate Hx, Hy
Return Δt
}
```

改良羅瑟梅爾模型並運用矢量法在二維空間平面模擬火勢蔓延

羅新恩* 蔣玫霞** 陳慶忠*** 吳文洪****

摘要

在全球氣候日益暖化的情況之下，森林大火對人類生命財產的威脅已經在世界各地成為一個嚴重的問題。一個可靠的野火模擬系統將可以有效的預測火勢蔓延並且能夠用來協助消防單位控制火區並且減少環境的破壞。電腦上模擬並且視覺化野火蔓延有兩種主要的研究類型：以柵格(raster)為基礎的展現方法和以矢量(vector)為基礎的展現方法。兩種方法各有他們的優點和缺失。以矢量為基礎的展現方法有一個最大的挑戰是不同擴張面交錯融合(crossover)的問題。在 1998 年 Osher 和 Sethian [1] 發表水平集法(level set method)的概念之前，所有對擴張面交錯融合問題的解決方法都需要煩瑣且耗時的計算。水平集方法是眾所周知的，能夠輕易的處理各種拓撲結構變化的問題，例如合併或撕裂接口。因此以矢量為基礎的展現方法來模擬火勢蔓延的模擬系統就可以利用水平集方法來解決野火蔓延交錯時的問題。第一位利用水平集法的概念來建立野火蔓延模擬系統的研究學者是 V. Mallet, D. Keyes, and F. Fendell [2]。他們利用惠更斯(Huygens)原理和橢圓形擴散假設來計算野火蔓延的距離和方向、然後結合水平集法的概念來擴大野火的蔓延。2009 年他們發表的論文中顯示出在控制風勢的情況下表現的完美橢圓火形。但是野火的蔓延還需要考慮地形地貌等的複雜環境。在這篇研究中、我們希望利用羅瑟梅爾火勢擴散的數學模型運用在水平集方法中來建立一個更適合自然環境的野火模擬系統。羅瑟梅爾火勢擴散的數學模型是美國用在建立野火模擬系統最廣泛的一個數學公式。我們的目標是將羅瑟梅爾模型分割成燃料、風勢和地形三個部分。將這三個部分分別利用水平集法來計算野火蔓延的距離，再將其結合算出最終的距離和方向。因為羅瑟梅爾模型是一維空間的擴散公式，我們還必需利用惠更斯原理和橢圓形擴散假設來計算野火蔓延的方向使之可以用來計算二維空間的擴展。

關鍵字：羅瑟梅爾模型、 矢量空間方法、 水平集法。

* 醒吾科技大學 資訊管理系 副教授
** 醒吾科技大學 資訊管理系 副教授
*** 醒吾科技大學 資訊管理系 助理教授
**** 醒吾科技大學 財務金融系 助理教授