

## **IS THE S&P RATING A GOOD CREDIT RISK PROXY? A STOCHASTIC FRONTIER APPROACH**

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### **ABSTRACT**

We use a stochastic frontier model with firm-specific technical inefficiency effects in a panel framework (Battese and Coelli, 1995) to assess the default probability (DP) based on the discrete-time hazard model (DHM; Shumway, 2001) and the long-term issuer credit rating (LTR) provided by Standard and Poor's Ratings Services (S&P). For studying the effect of S&P LTR on technical inefficiency, a special design matrix is used to represent the ordinal level of S&P LTR in the stochastic frontier model. For US industrial firms spanning the period 1998-2011 on a monthly basis, our empirical result shows that S&P LTR provides significantly more information content about firm's technical inefficiency than DP based on DHM. Combining the result with the fact that economic-based efficiency measures are reasonable indicators of the long-term health and prospects of firms (Baek and Pagán, 2002), we conclude that S&P LTR is a good credit risk proxy of firms.

Key words and phrases: Discrete-time hazard model, Expanding rolling window approach, Long-term issuer credit rating, Robust Wald test, Stochastic frontier model.

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## 1. Introduction

Credit rating is a type of ranking for credit risk, and the investor frequently uses it as a surrogate measure for the creditworthiness of bonds and companies. Under the New Basel Capital Accord (Basel III), it will play an even more important role in capital markets than it has so far. Currently, there are many widely recognized credit rating agencies, such as Standard and Poor's Ratings Services (S&P), Moody's Investors Service, Fitch Ratings Ltd., etc. They routinely provide the bond rating and the issuer credit rating. The bond rating attempts to measure likelihood of the default or delayed payment of a bond issue. The issuer credit rating is an overall assessment of the creditworthiness of a company. This study focuses on the S&P long-term issuer credit rating (LTR). According to the definition given by S&P, the LTR focuses on the obligor's capacity and willingness to meet its long-term financial commitments as they come due.

Default prediction has been routinely studied by academics, practitioners, and regulators. It is an important tool for credit risk management. The well-known prediction models include discriminant analysis model (Altman, 1968), Merton model (Merton, 1974; Vassalou and Xing, 2004), logit model (Ohlson, 1980), and probit model (Zmijewski, 1984), to name only a few. The common principal of these approaches is that the models are developed using single-period firm data. Shumway (2001) points out that such prediction processes are static in nature, since they ignore the changing characteristics of firms through time. In order to avoid possible loss of prediction power due to using static models, Shumway (2001) and Chava and Jarrow (2004) propose the discrete-time hazard model (DHM) using multiple-period firm data for default prediction. Their novel model applies the idea of survival analysis (Cox and Oakes, 1984), and has the advantage of using all available information of firms to build up a prediction system so that each firm's default risk at each time point can be determined. Thus, DHM is a dynamic forecasting model. Shumway (2001) has proved that DHM has better prediction performance than the above static models.

The performance of default models is mainly assessed in the literature by performing prediction-oriented tests; see for example, the above references. Recently, Hwang et al. (2011) suggest using information content of technical inefficiency to assess the default models. Their results show that the default probability (DP) based on DHM (DP-DHM) provides significantly more information than that based on Merton model. On the other hand, Hillegeist et al. (2004) show that DP based on Merton model provides significantly more information content about credit risk than other static models. Hence, these results in terms of prediction-oriented or information content tests indicate that DP-DHM is a better credit risk proxy than that based on the static model.

The DP-DHM is only based on public information. In contrast, the S&P LTR is based on both public information and private information conveyed to the rating agencies by firm. Both DP-DHM and S&P LTR are credit risk proxies of firms. In

this paper, we use information content of firm's technical inefficiency to assess them. For doing so, a stochastic frontier model with firm-specific technical inefficiency effects in a panel framework (Battese and Coelli, 1995) is considered. A firm is characterized as technically inefficient if it is not able to reach maximum output given its available resources and technology.

Stochastic frontier analysis is a method of economic modeling. It has been widely used to estimate technical inefficiencies of production of firms; see for example, the monographs by Kumbhakar and Lovell (2003) and Coelli et al. (2005). It also has been used in the studies of corporate financial decision and bank efficiency; see for example, Hunt-McCool et al. (1996), Baek (2004), Fries and Taci (2005), Lensink et al. (2008), and Kauko (2009).

To compare DP-DHM and S&P LTR on their abilities in explaining technical inefficiency, they are separately used in the stochastic frontier model. In the model, the explanatory variables for technical inefficiency effects are taken as the DP-DHM (or S&P LTR) and control variable Age<sup>1</sup>. Since S&P begins to use the term LTR on September 1, 1998 (Standard and Poor, 2002, pp. 622-623), we collect S&P LTRs from the COMPUSTAT database during the period from September 1998 to December 2011. Using an expanding rolling window approach (Hillegeist et al., 2004; Chava et al., 2011), we compute out-of-sample values of DP-DHM during the period from September 1998 to December 2011. For studying the effect of S&P LTR on technical inefficiency, a special design (Hwang et al., 2012) is used to represent the ordinal level of S&P LTR in all analysis. Using the special design, we can examine whether the worse the level of S&P LTR, the less the technical efficiency. If the result is statistically confirmed, then we are confident that the worse the level of S&P LTR, the worse the creditworthiness of the issuer.

After deriving those values of DP-DHM and S&P LTR, the stochastic frontier model is estimated for the period from September 1998 to December 2011. The studied data are collected from both COMPUSTAT and CRSP databases. Our final sample for estimating the stochastic frontier model comprises 1,501 firms with 91,729 firm-month observations. After controlling for the firm's age, three important results based on our empirical results are obtained. First, if a firm has a higher DP, then it is less technically efficient. Second, a firm with a worse level of S&P LTR generally has less technical efficiency. Finally, S&P LTR provides significantly more information content about firm's technical inefficiency than DP-DHM. The last result indicates a potential increase of information content by using S&P LTR instead of DP-DHM as the credit risk proxy of firms. By these empirical results and the fact that economic-based efficiency measures are reasonable indicators of the long-term health and prospects of firms (Baek and Pagán, 2002), we conclude that S&P LTR is a better credit risk proxy of firms than DP-DHM.

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<sup>1</sup>The variable Age denotes a firm's age which is defined as the number of months it has been listed on the NYSE / AMEX / NASDAQ (Shumway, 2001).

The paper is organized as follows. Section 2 introduces a design matrix of S&P LTR, and the formulations of DHM and the stochastic frontier model with firm-specific technical inefficiency effects. Section 3 describes the data under study. Section 4 presents our empirical results. Finally, conclusions are given in Section 5.

## 2. Methodology

In this section, we first give the formulation of the design matrix that is used to represent the level of S&P LTR. Then we give the basic idea of DHM. Afterward we introduce the stochastic frontier model with firm-specific technical inefficiency effects.

### 2.1 The Design Matrix of S&P LTR

The level of S&P LTR is an ordinal variable, and ranges from AAA to D. Since S&P does not assign the level C to any firm in the COMPUSTAT database, we exclude the C level in the analysis (Standard and Poor, 2002, pp. 624-625). We use the discrete variable  $R$  to express the nine levels of S&P LTR; see Table 1. The value of  $R$  ranges from 1 to 9. The larger the value of  $R$ , the worse the creditworthiness of the issuer. Thus, the values of  $R = 1, \dots, 9$  represent the levels AAA, ..., D of S&P LTR, respectively. We use the eight dummy variables  $(T_1, \dots, T_8)^T$  to represent the value of  $R$ . Define the  $k$ -th dummy variable as follow:

$$T_k = I(R > k), \quad (1)$$

for each  $k = 1, \dots, 8$ , where the upper index  $T$  stands for the transpose of the associated matrix and  $I(\bullet)$  is an indicator function. Using the eight dummy variables, the design matrix of S&P LTR is defined in Table 1. Table 1 shows that if  $R = j$ , the associated level of S&P LTR is expressed as an  $8 \times 1$  vector with the first  $j - 1$  component as 1 and all other components as 0, for each  $j = 2, \dots, 9$ . Note that if  $R = 1$  then the corresponding level of S&P LTR is represented by an  $8 \times 1$  vector with all components as 0. Using this design matrix, we can examine whether a firm with a worse level of S&P LTR attains the less technical efficiency. The details are given in the last paragraph of Section 2.3.

### 2.2 DHM

The DHM is constructed using multiple-period firm data and the idea of survival analysis. It has been proved by many examples to be a useful prediction model (Hillegeist et al., 2004; Campbell et al., 2008; Hwang et al., 2011). The panel data used in DHM are determined by two factors: the sampling period and the sampling criterion. The sampling criterion is that all industrial firms listed on the NYSE/AMEX/NASDAQ during the sampling period are recruited in the sample. All information occurred at the

discrete time points during the sampling period is collected from both COMPUSTAT and CRSP databases. Let the sampling period be  $[\xi_1, \xi_2]$ . Suppose that there are  $n$  selected companies under the particular sampling scheme. We denote the panel data by

$$\{(Y_{i,j}, x_{i,j}) : j = s_i, \dots, t_i, i = 1, \dots, n\}.$$

Here  $s_i$  denotes the first observation time and  $t_i$  the last observation time for the  $i$ -th firm in the sampling period  $[\xi_1, \xi_2]$ . At the last observation time  $t_i$ ,  $Y_{i,t_i} = 1$  indicates that the  $i$ -th company is in the default status, and  $Y_{i,t_i} = 0$  otherwise. At the observation time  $j < t_i$ , we always have  $Y_{i,j} = 0$ . Finally, we let  $x_{i,j}$  be the value of the  $d \times 1$  firm-specific predictor  $X$  collected from the  $i$ -th firm at time  $j$ .

The DHM is formally defined using the log-likelihood function of the panel data  $\{(Y_{i,j}, x_{i,j}) : j = s_i, \dots, t_i, i = 1, \dots, n\}$ . The log-likelihood function of the panel data can be expressed as:

$$\ell = \sum_{i=1}^n \sum_{j=s_i}^{t_i} Y_{i,j} \log[h(j, x_{i,j})/\{1 - h(j, x_{i,j})\}] + \sum_{i=1}^n \sum_{j=s_i}^{t_i} \log\{1 - h(j, x_{i,j})\}, \quad (2)$$

where

$$h(j, x_{i,j}) = \exp(\alpha + \beta x_{i,j})/\{1 + \exp(\alpha + \beta x_{i,j})\}. \quad (3)$$

Here  $\alpha$  and  $\beta = (\beta_1, \dots, \beta_d)$  are  $1 \times 1$  and  $1 \times d$  vectors of parameters, respectively. The result of (2) has been given in (21) of Allison (1982). The quantity  $h(j, x_{i,j})$  is the value of hazard function representing DP happening instantly at time  $j$  for the  $i$ -th company which is non-default before time  $j$  for each  $j = s_i, \dots, t_i$  and  $i = 1, \dots, n$ .

Let  $\hat{\alpha}$  and  $\hat{\beta}$  be the maximum likelihood estimates of parameters  $\alpha$  and  $\beta$  in (3). Using (3) and replacing the unknown parameters  $\alpha$  and  $\beta$  with their maximum likelihood estimates  $\hat{\alpha}$  and  $\hat{\beta}$ , if a firm has the predictor value  $x_0$  at time  $t_0$ , then its predicted instant DP-DHM can be given by:

$$\hat{h}(t_0, x_0) = \exp(\hat{\alpha} + \hat{\beta}x_0)/\{1 + \exp(\hat{\alpha} + \hat{\beta}x_0)\}. \quad (4)$$

### 2.3 The Stochastic Frontier Model

We use the stochastic frontier model proposed by Battese and Coelli (1995) to examine the relationship between DP-DHM (or S&P LTR) and technical inefficiency for firms in this paper. Using a translog specification, the model is given by:

$$\ln(Q_{i,t}) = (\gamma_0 + \gamma_1 w_{i,t}) + \sum_{j=1}^m (\eta_{0,j} + \eta_{1,j} w_{i,t}) \text{SIC}_j + V_{i,t} - U_{i,t}, \quad (5)$$

for  $t = 1, \dots, t_i$  and  $i = 1, \dots, n$ . Here  $\ln(Q_{i,t})$  is the natural logarithm of the dollar value of production  $Q_{i,t}$  for the  $i$ -th firm at time  $t$ , and  $Q_{i,t}$  is taken as the sum of sales and inventory increase during the period from the time point  $t - 1$  to  $t$ ;

$w_{i,t} = \{\ln(L_{i,t}), \ln(K_{i,t}), \ln(L_{i,t})^2, \ln(K_{i,t})^2, \ln(L_{i,t}) \ln(K_{i,t})\}^T$  is a  $5 \times 1$  vector of explanatory variables related to the stochastic frontier production function for the  $i$ -th firm at time  $t$ ;

$\ln(L_{i,t})$  is the natural logarithm of the total number of employees for the  $i$ -th firm at time  $t$ ;

$\ln(K_{i,t})$  is the natural logarithm of the value of fixed assets for the  $i$ -th firm at time  $t$ ;

$SIC_j$  is an industry dummy variable, for each  $j = 1, \dots, m$ , which is designed for capturing differences in the production technology across  $m + 1$  industry categories<sup>2</sup>;

$V_{i,t}$ s are independent and identically distributed  $N(0, \sigma_v^2)$  random errors and independently distributed of  $U_{i,t}$ s;

$U_{i,t}$ s are nonnegative random variables, associated with technical inefficiency effects, which are assumed to be independently distributed, such that  $U_{i,t}$  is obtained by truncation (at zero) of the normal distribution with mean  $\delta_0 + \delta_1 \text{Age}_{i,t} + \delta_2 z_{i,t}$  and variance  $\sigma_u^2$ ;

$z_{i,t}$  is a  $1 \times 1$  (or  $8 \times 1$ ) vector, and consists of the out-of-sample DP-DHM (or the eight dummy variables of S&P LTR) for the  $i$ -th firm at time  $t$ ;

$\gamma_0, \gamma_1, \eta_{0,j}, \eta_{1,j}, \delta_0, \delta_1$ , and  $\delta_2$  are unknown  $1 \times 1, 1 \times 5, 1 \times 1, 1 \times 5, 1 \times 1, 1 \times 1$ , and  $1 \times 1$  (or  $1 \times 8$ ) vectors of parameters to be estimated, respectively, for each  $j = 1, \dots, m$ .

The unknown parameter vector  $\theta = (\gamma_0, \gamma_1, \eta_{0,1}, \dots, \eta_{0,m}, \eta_{1,1}, \dots, \eta_{1,m}, \delta_0, \delta_1, \delta_2)$  and variance parameters  $\sigma^2 = \sigma_v^2 + \sigma_u^2$  and  $\zeta = \sigma_u^2 / \sigma^2$  in (5) can be estimated by maximizing the log-likelihood function of the panel data (Battese and Coelli, 1993). Their maximum likelihood estimates  $\hat{\theta} = (\hat{\gamma}_0, \hat{\gamma}_1, \hat{\eta}_{0,1}, \dots, \hat{\eta}_{0,m}, \hat{\eta}_{1,1}, \dots, \hat{\eta}_{1,m}, \hat{\delta}_0, \hat{\delta}_1, \hat{\delta}_2), \hat{\sigma}^2$ , and  $\hat{\zeta}$  can be simply computed using the free software FRONTIER Version 4.1 written by Professor Tim Coelli and available at the website <http://www.uq.edu.au/economics/cepa/frontier.htm>. On the other hand, to allow for non-independence of technical inefficiency over time, the Wald test using the robust standard error of estimated parameter (Alvarez et al., 2006) is employed to test the significance of parameter in the empirical study.

If  $z_{i,t}$  is the vector of the eight dummy variables of S&P LTR, then  $\hat{\delta}_2 = (\hat{\delta}_{2,1}, \dots, \hat{\delta}_{2,8})$ . Using the value of  $\hat{\delta}_2 = (\hat{\delta}_{2,1}, \dots, \hat{\delta}_{2,8})$ , if S&P LTR is a good credit risk proxy, then each element in  $\hat{\delta}_2$  should be positive. In this case, the worse the level of S&P LTR, the larger the corresponding value of  $\hat{\delta}_2 z_{i,t}$ , and thus the lower the estimated technical efficiency.

<sup>2</sup>Seven industry categories are defined in Table 4 of Section 3 for our empirical studies.

### 3. Data and Variable Estimation

The predictors in DHM for computing out-of-sample DP-DHM are taken as the accounting and market-driven variables suggested by Campbell et al. (2008). Their definitions are given in Table 2. The sampling period of these predictors is between January 1991 and November 2011. Default filings between February 1991 and December 2011 are identified from the CRSP database according to the delisting codes 400~490, 572, and 574. We also identify a delisting as default if the delisting reason is 02 or 03 in the COMPUSTAT database (Duffie et al., 2007). The sampled firms consist of all industrial firms listed on the NYSE / AMEX / NASDAQ, and have complete predictor values for calculating DP-DHM. The predictor values are collected from both COMPUSTAT and CRSP databases. The financial services firms with Standard Industrial Classification (SIC) codes 6000~6999 are eliminated from the sample due to the unique capital requirements and regulatory structure in that industry group. Since the financial statements are usually released several months after the reporting period, we lag all the accounting items by three months (Hillegeist et al., 2004). Our final sample for computing out-of-sample DP-DHM consists of 10,578 solvent firms, 744 default firms, and 1,083,867 firm-month observations. In order to eliminate the influence of outliers, the values of each predictor (except the predictor PRICE) are winsorized using a 5/95 percentile interval (Campbell et al., 2008). The summary statistics of the predictor values are given in Table 3.

Based on the above panel dataset, we calculate out-of-sample DP-DHM using an expanding rolling window approach. For the first window, we estimate updated coefficients of DHM using the firm-month data from January 1991 to July 1998 and default outcomes from February 1991 to August 1998. These updated coefficients are combined with predictor values in August 1998 to evaluate out-of-sample DP-DHM in September 1998. For the second window, we use the firm-month data from January 1991 to August 1998 and default outcomes from February 1991 to September 1998 to estimate the second set of updated coefficients of DHM. This second set of coefficients is combined with predictor values in September 1998 to produce out-of-sample DP-DHM in October 1998. The process is continued so that the last set of updated coefficients of DHM used to generate out-of-sample DP-DHM in December 2011 is based on the firm-month data from January 1991 to October 2011 and default outcomes from February 1991 to November 1998. Hence, we collect out-of-sample DP-DHM from September 1998 to December 2011. The maximum likelihood estimates of parameters in DHM for computing out-of-sample DP-DHM in each window are given in Figure 1. Figure 1 shows that, in each window, most of the predictors in DHM have significant effects on DP at the 5% level, and the values of their estimated coefficients are of expected signs. The estimation results in Figure 1 indicate that most of the variables in DHM are effective predictors for default prediction.

Figure 1 also shows that the estimated coefficients are stable between 2003 and 2009, but most of them behave different around 2009, when the financial crisis occurs. The result indicates that the financial crisis may cause a change point problem to DHM. In order to alleviate the effect of this potential pitfall, we may apply the idea of the varying-coefficient model (Fan and Zhang, 2008) to DHM and treat the constant coefficients of hazard function in DHM as the functions of macroeconomic variables. The resulting model is more general and flexible in modeling the hazard function than DHM, and is estimated using the local maximum likelihood method (Hwang, 2012).

To study the relationship between DP-DHM (or S&P LTR) and technical inefficiency for firms, we collect panel data from the COMPUSTAT database to build the stochastic frontier model during the period from September 1998 to December 2011. The out-of-sample DP-DHM (or S&P LTR) and the control variable Age are taken as the explanatory variables related to technical inefficiency effects in the stochastic frontier model. After deleting observations with missing values, our final sample for estimating the stochastic frontier model consists of 1,501 firms with 91,729 firm-month observations.

Table 4 shows the definitions of industry dummy variables and the frequency distributions of industry categories corresponding to the industry dummy variables. The industry dummy variables are included for studying industry effects on technical inefficiency of firms by capturing differences in the production technology across seven industry categories<sup>3</sup>. Table 5 reports summary statistics for the predictor values employed in the stochastic frontier model. Table 6 shows the frequency distributions of S&P LTR.

## 4. Empirical Results

The empirical results based on the stochastic frontier model with different specifications of the inefficiency effect equation are given in Table 7. In Table 7, Model 1 uses DP-DHM and Age as explanatory variables, and Model 2 employs S&P LTR and Age as explanatory variables in the inefficiency effect equation. Panel A of Table 7 shows that most of parameters of the production function under each of Models 1 and 2 are statistically significant at the 5% level. The result of statistical significance for most of industry dummy variables indicates that the production technology indeed varies across industries. The estimated coefficients of the production function under each of Models 1 and 2 are used to calculate marginal products of the two inputs, labor and fixed assets, for each sample point. The average values of the two marginal products

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<sup>3</sup>The industries are classified by the first digit of the four-digit SIC code. There are one company with SIC code less than 1000 and eight companies with SIC code greater than 9000. These nine companies are dropped from the sample. If such industries with few companies are included in the model, then the resulting number of industry indicator variables is increased. Thus, the estimates of parameters in the corresponding model might become less precise.

under Model 1 are 0.510 and 0.357, and those under Model 2 are 0.486 and 0.348 for the inputs labor and fixed assets, respectively in each case. By the positive signs of the two marginal products, both Models 1 and 2 appear to attain the implications in economics. Hence, the estimated coefficients of the production function under each of Models 1 and 2 have appropriate signs.

Panel B of Table 7 shows that the estimated coefficient of the control variable Age under each of Models 1 and 2 is negative and statistically significant at the 5% level, which indicates that the old firms are more technically efficient than the young ones. After controlling for the firm's age, DP-DHM is positively correlated with technical inefficiency since the sign of its coefficient is positive at the 1% level of significance. The result indicates that if a firm has a higher DP, then it is less technically efficient. Under Model 2, the coefficient estimates of the dummy variables  $T_2$ ,  $T_3$ , and  $T_7$  are negative, but those of  $T_3$  and  $T_7$  are non-significant at the 10% level. The coefficient estimates of the other dummy variables are all positive, and those of  $T_4$ ,  $T_5$ , and  $T_6$  are significant at the 1% level. By the signs of coefficient estimates of the eight dummy variables  $T_1, \dots, T_8$ , the relationship between the worse level of S&P LTR and the lower technical efficiency is significant, but the relationship between the worse level of S&P LTR and the larger technical efficiency is non-significant, except for the level A.

Panel C of Table 7 presents that the estimated ratios of variance parameter  $\zeta$  are 0.935 and 0.913 under Models 1 and 2, respectively. Both of them are statistically significant at the 1% level. The result indicates that the inefficiency effect is likely to be important because most of the composite error variance  $\sigma^2$  is accounted for by the variance of the inefficiency effect in each model. Further, by performing a likelihood-ratio test, the null hypothesis that the values of parameters in inefficiency effect equation and  $\zeta$  are all equal to zero is rejected at the 1% level of significance under each of Models 1 and 2. The test result is given in Panel D of Table 7, and indicates that the inefficiency frontier model is appropriate under each model.

Table 7 also shows that S&P LTR provides more information content about firm's technical inefficiency than DP-DHM. This superiority may be directly observed from Panel D of Table 7 in the perspective that Model 2 has a larger log-likelihood statistic and a smaller value of Akaike information criterion (AIC) than Model 1. To test the null hypothesis that the information content about firm's technical inefficiency provided by Model 2 is not more than that generated by Model 1, a non-nested hypothesis test procedure in Vuong (1989) based on the log-likelihood statistic is performed. Since DP-DHM and S&P LTR are separately used in the inefficiency effect equation, their corresponding Models 1 and 2 are non-nested models. Panel E of Table 7 shows that the null hypothesis of interest was rejected at the 1% level of significance. The test result indicates that the information content about firm's technical inefficiency provided by S&P LTR is significantly more than that generated by DP-DHM at the 1% level of significance.

Overall, the results in Table 7 show that S&P LTR has the advantage of having better ability in explaining firm's technical inefficiency over DP-DHM. Combining the result with the fact that economic-based efficiency measures are reasonable indicators of the long-term health and prospects of firms, we conclude that S&P LTR is a better credit risk proxy than DP-DHM.

## 5. Conclusions

In this paper, DP-DHM is compared with S&P LTR on their information content about firm's technical inefficiency. For doing so, we compute out-of-sample values of DP-DHM for the period from September 1998 to December 2011 using an expanding rolling window approach. We also collect S&P LTRs from the COMPUSTAT database during the period from September 1998 to December 2011. After deriving those values of DP-DHM and S&P LTRs, they are separately used in the stochastic frontier model with firm-specific technical inefficiency effects. The model is estimated for the period from September 1998 to December 2011. The studied data are collected from both COMPUSTAT and CRSP databases. Our final sample for estimating the stochastic frontier model consists of 1,501 firms with 91,729 firm-month observations.

After controlling for the firm's age, three important results based on our empirical studies are obtained. First, DP-DHM is significantly positively correlated with technical inefficiency. The result indicates that if a firm has a higher DP, then it is less technically efficient. Second, the relationship between the worse level of S&P LTR and the lower technical efficiency is significant, but the relationship between the worse level of S&P LTR and the larger technical efficiency is non-significant, except for the level A. Finally, S&P LTR provides significantly more information content about firm's technical inefficiency than DP-DHM. By our empirical results and the fact that economic-based efficiency measures are reasonable indicators of the long-term health and prospects of firms, we conclude that S&P LTR is a better credit risk proxy than DP-DHM.

Table 1. The design matrix for the nine levels of S&amp;P LTR

S&P LTR	AAA	AA	A	BBB	BB	B	CCC	CC	D
$R$	1	2	3	4	5	6	7	8	9
$T_1$	0	1	1	1	1	1	1	1	1
$T_2$	0	0	1	1	1	1	1	1	1
$T_3$	0	0	0	1	1	1	1	1	1
$T_4$	0	0	0	0	1	1	1	1	1
$T_5$	0	0	0	0	0	1	1	1	1
$T_6$	0	0	0	0	0	0	1	1	1
$T_7$	0	0	0	0	0	0	0	1	1
$T_8$	0	0	0	0	0	0	0	0	1

We use the discrete variable  $R$  to denote the level of S&P LTR. The value of  $R$  ranges from 1 to 9. The eight dummy variables  $(T_1, \dots, T_8)^T$  are used to represent the values of  $R$ .

The  $k$ -th dummy variable  $T_k$  is defined by  $T_k = I(R > k)$ , for each  $k = 1, \dots, 8$ , where the upper index  $T$  stands for the transpose of the associated matrix and  $I(\cdot)$  is an indicator function. Since S&P does not assign the level C to any firm in the COMPUSTAT database, we exclude the C level in the analysis (Standard and Poor, 2002, pp. 624-625).

Table 2. The definitions of Campbell's eight predictors

Variable	Definition
NIMTAAVG	$NIMTAAVG = (1 - \omega^3)(1 - \omega^{12})^{-1}(NIMTA_4 + \omega^3 NIMTA_3 + \omega^6 NIMTA_2 + \omega^9 NIMTA_1)$ , the weighted average of four quarterly $NIMTA$ with $\omega = 2^{-1/3}$ and $NIMTA$ as net income divided by market-valued total assets*
TLMTA	Total liabilities divided by market-valued total assets*
EXRETAVG	$EXRETAVG = (1 - \omega)(1 - \omega^{12})^{-1}(EXRET_{12} + \dots + \omega^{11} EXRET_1)$ , the weighted average of twelve monthly excess returns ( $EXRET$ ) with $\omega = 2^{-1/3}$ and $EXRET$ as the return on the firm minus the value-weighted CRSP index return
SIGMA	Annualized square root of the average of the squared deviations in the firm's daily stock returns from zero over the past three months
RSIZE	Logarithm of each firm's market equity value divided by the total CRSP market equity value
CASHMTA	Cash and short-term investments divided by market-valued total assets*
MB	Market equity value divided by book equity value
PRICE	Logarithm of stock price if the price is below \$15, and logarithm of \$15 otherwise

\*The market-valued total assets is the sum of the book value of liabilities and the market value of equities.

Table 3. Summary statistics of the panel data used to compute out-of-sample DP-DHM

Predictor	Mean	Median	Standard deviation	Minimum	Maximum
NIMTAAVG	-0.004	0.005	0.025	-0.075	0.023
TLMTA	0.350	0.307	0.246	0.029	0.842
EXRETAVG	-0.010	-0.005	0.051	-0.121	0.082
SIGMA	0.620	0.527	0.353	0.197	1.492
RSIZE	-10.932	-10.996	1.873	-14.096	-7.363
CASHMTA	0.102	0.054	0.117	0.002	0.425
MB	2.359	1.859	1.695	0.440	7.003
PRICE	1.982	2.327	0.895	-4.159	2.708

The predictors in DHM are the accounting and market-driven variables suggested by Campbell et al. (2008). The sampled firms consist of all industrial firms listed on the NYSE/AMEX/NASDAQ, and have complete predictor values for calculating the DP-DHM. The predictor values are collected from both COMPUSTAT and CRSP databases during the period from January 1991 to November 2011. Default filings between February 1991 and December 2011 are identified from the COMPUSTAT or CRSP database. In order to eliminate the influence of outliers, the values of each predictor (except the predictor PRICE) are winsorized using a 5/95 percentile interval (Campbell et al., 2008). Our final sample for computing out-of-sample DP-DHM consists of 10,578 solvent firms, 744 default firms, and 1,083,867 firm-month observations.

Table 4. The definitions of industry dummy variables and the frequency distributions of the sampled companies

Industry dummy variable	SIC code	Industry	Frequency
SIC <sub>1</sub>	1000~1999	Mining, Construction	154
SIC <sub>2</sub>	2000~2999	Food, Tobacco, Textile, and Paper Manufacturing etc.	300
SIC <sub>3</sub>	3000~3999	Rubber, Leather, Stone, and Industrial Manufacturing etc.	427
SIC <sub>4</sub>	5000~5999	Wholesale Trade, Retail Trade	173
SIC <sub>5</sub>	7000~7999	Hotel, Personal, Business, and Automotive Repair Services etc.	173
SIC <sub>6</sub>	8000~8999	Health, Legal, Education, and Social Services etc.	59
Reference level	4000~4999	Transportation, Communications, Electric, Gas, and Sanitary Services	215
Total firms			1,501

The industries are classified by the first digit of the four-digit SIC code, and are used in the stochastic frontier model for capturing differences in the production technology across seven industry categories. The SIC codes 4000~4999 are used as the reference level for studying the industry effects in stochastic frontier model. The financial services firms with SIC codes 6000~6999 are eliminated from the sample due to the unique capital requirements and regulatory structure in that industry group. There are one company with SIC code less than 1000 and eight companies with SIC code greater than 9000. These nine companies are dropped from the sample. If such industries with few companies are included in the model, then the resulting number of industry indicator variables is increased. Thus, the estimates of parameters in the corresponding model might become less precise.

Table 5. Summary statistics of the panel data used to estimate the stochastic frontier model

Variable	Mean	Median	Standard deviation	Minimum	Maximum
$\ln(Q)$	6.5927	6.4837	1.6180	-4.0745	11.7082
$\ln(L)$	2.3293	2.3125	1.6357	-5.8091	7.6497
$\ln(K)$	6.6284	6.5120	1.8461	-1.3863	12.2768
Age	286.8221	186.0000	259.0525	2.0000	1033.0000
DP-DHM	0.0003	0.0000	0.0036	0.0000	0.2794

The values of the variables  $\ln(Q)$ ,  $\ln(L)$ , and  $\ln(K)$  are collected from the COMPUSTAT database during the period from September 1998 to December 2011. Here  $\ln(Q)$  is the natural logarithm of the dollar value of production and  $Q$  is taken as the sum of sales and inventory increase for a month,  $\ln(L)$  is the natural logarithm of the total number of employees, and  $\ln(K)$  is the natural logarithm of the value of fixed assets. The values of DP-DHM are produced for the period from September 1998 to December 2011 using an expanding rolling window approach. The variable Age denotes a firm's age which is defined as the number of months it has been listed on the NYSE/AMEX/NASDAQ. Our final sample for estimating the stochastic frontier model consists of 1,501 firms with 91,729 firm-month observations.

Table 6. The frequency distributions of the panel data for studying the relationship between S&amp;P LTR and technical inefficiency according to S&amp;P LTR.

S&P LTR	$R$	Frequency
AAA	1	883
AA	2	3,403
A	3	14,305
BBB	4	24,531
BB	5	27,290
B	6	19,375
CCC	7	1,666
CC	8	127
D	9	149
Total firm-months		91,729

Table 7. Maximum likelihood estimates of parameters in the stochastic frontier model with different specifications of the inefficiency effect equation

Variable	Model 1		Model 2	
	Coefficient	Robust standard error	Coefficient	Robust standard error
Panel A: Production function				
Intercept	4.648***	0.629	4.914***	0.651
$\ln(L)$	1.040***	0.201	1.051***	0.203
$\ln(K)$	-0.314	0.225	-0.376	0.229
$\ln(L)^2$	0.021*	0.012	0.019	0.012
$\ln(K)^2$	0.062***	0.019	0.067***	0.019
$\ln(L)\ln(K)$	-0.089***	0.031	-0.093***	0.031
SIC <sub>1</sub>	3.908***	0.668	3.675***	0.693
SIC <sub>2</sub>	0.080	0.870	0.053	0.884
SIC <sub>3</sub>	-0.186	0.830	-0.285	0.865
SIC <sub>4</sub>	0.901	0.993	0.732	1.003
SIC <sub>5</sub>	-0.123	0.767	-0.354	0.796
SIC <sub>6</sub>	-3.711***	1.203	-3.825***	1.286
$\ln(L)$ SIC <sub>1</sub>	0.373	0.237	0.166	0.239
$\ln(L)$ SIC <sub>2</sub>	-0.297	0.320	-0.398	0.313
$\ln(L)$ SIC <sub>3</sub>	-0.249	0.324	-0.301	0.324
$\ln(L)$ SIC <sub>4</sub>	-0.947***	0.367	-1.100***	0.387
$\ln(L)$ SIC <sub>5</sub>	-0.594**	0.243	-0.695***	0.249
$\ln(L)$ SIC <sub>6</sub>	-0.200	0.311	-0.288	0.357
$\ln(K)$ SIC <sub>1</sub>	-1.032***	0.246	-0.917***	0.251
$\ln(K)$ SIC <sub>2</sub>	0.055	0.324	0.079	0.320
$\ln(K)$ SIC <sub>3</sub>	0.244	0.333	0.275	0.339
$\ln(K)$ SIC <sub>4</sub>	0.329	0.425	0.446	0.432
$\ln(K)$ SIC <sub>5</sub>	0.461	0.285	0.552	0.292
$\ln(K)$ SIC <sub>6</sub>	1.679***	0.508	1.755***	0.541
$\ln(L)^2$ SIC <sub>1</sub>	0.034*	0.020	0.032	0.021
$\ln(L)^2$ SIC <sub>2</sub>	-0.007	0.041	-0.003	0.038
$\ln(L)^2$ SIC <sub>3</sub>	0.007	0.042	0.015	0.041
$\ln(L)^2$ SIC <sub>4</sub>	-0.190***	0.025	-0.186***	0.026
$\ln(L)^2$ SIC <sub>5</sub>	-0.019	0.018	-0.023	0.018
$\ln(L)^2$ SIC <sub>6</sub>	-0.097**	0.041	-0.084*	0.043

(continued on next page)

Table 7 (continued)

Variable	Model 1		Model 2	
	Coefficient	Robust standard error	Coefficient	Robust standard error
$\ln(K)^2 \text{SIC}_1$	0.065***	0.021	0.054**	0.022
$\ln(K)^2 \text{SIC}_2$	0.006	0.030	0.002	0.029
$\ln(K)^2 \text{SIC}_3$	-0.030	0.034	-0.033	0.034
$\ln(K)^2 \text{SIC}_4$	-0.064	0.046	-0.077*	0.047
$\ln(K)^2 \text{SIC}_5$	-0.074***	0.025	-0.083***	0.026
$\ln(K)^2 \text{SIC}_6$	-0.174***	0.049	-0.182***	0.052
$\ln(L)\ln(K) \text{SIC}_1$	-0.043	0.037	-0.015	0.037
$\ln(L)\ln(K) \text{SIC}_2$	0.027	0.062	0.042	0.059
$\ln(L)\ln(K) \text{SIC}_3$	0.052	0.070	0.055	0.068
$\ln(L)\ln(K) \text{SIC}_4$	0.257***	0.070	0.277***	0.073
$\ln(L)\ln(K) \text{SIC}_5$	0.128***	0.035	0.146***	0.036
$\ln(L)\ln(K) \text{SIC}_6$	0.088*	0.045	0.093*	0.050
Panel B: Inefficiency effect equation				
Intercept	-8.935***	0.970	-8.598***	3.301
Age	-0.008***	0.002	-0.003**	0.001
DP-DHM	44.248***	8.659		
T <sub>1</sub>			0.064	3.143
T <sub>2</sub>			-3.418***	0.942
T <sub>3</sub>			-0.030	0.823
T <sub>4</sub>			4.206***	0.959
T <sub>5</sub>			3.637***	0.761
T <sub>6</sub>			2.472***	0.478
T <sub>7</sub>			-2.575	2.189
T <sub>8</sub>			2.922	2.324
Panel C: Variance parameter				
$\sigma^2$	4.637***	0.248	3.475***	0.179
$\zeta$	0.935***	0.001	0.913***	0.001

(continued on next page)

Table 7 (continued)

Variable	Model 1		Model 2	
	Coefficient	Robust standard error	Coefficient	Robust standard error
Panel D: Model fit test				
Log-likelihood	-91539.370		-89843.911	
AIC	183172.740		179795.822	
Likelihood-ratio	6340.980***		9731.898***	
Panel E: Vuong test				
Z-statistic	21.411***			

Model 1 uses Age and DP-DHM as explanatory variables in the inefficiency effect equation. Model 2 employs Age and S&P LTR as explanatory variables in the inefficiency effect equation. Panels A and B show respectively the estimation results of production function and inefficiency effect equation in Models 1 and 2. Panel C presents the estimation results of variance parameters  $\sigma^2$  and  $\zeta$  in Models 1 and 2. To allow for non-independence of technical inefficiency over time, the Wald test using the robust standard error of estimated parameter is employed in each of Panels A, B, and C to test the significance of parameter. Panel D contains the result of model fit test for the inefficiency frontier model under Models 1 and 2. Panel E gives the Z-statistic of Vuong test. The notations \*\*\*, \*\*, and \* indicate the significance of test at the 1%, 5%, and 10% levels, respectively.

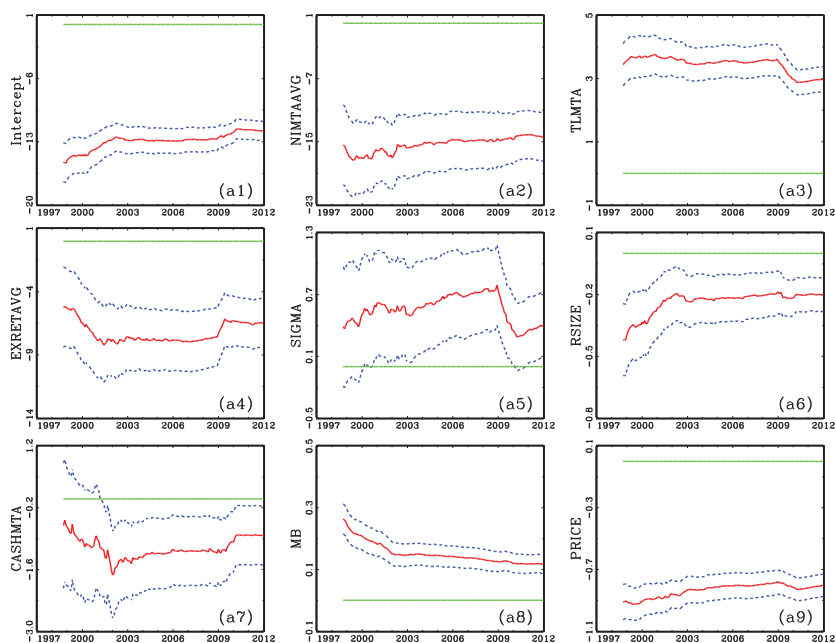


Figure 1. Plots of maximum likelihood estimates of parameters in DHM. These estimates are produced using the panel data collected in Section 3. The panel data are based on Campbell's eight predictors, collected during the period from January, 1991 to November, 2011, and composed of 10,578 solvent firms, 744 default firms, and 1,083,867 firm-month observations. In each panel, the dashed and the solid curves stand for the 95% confidence interval and the estimate of the associated parameter, respectively. Also, the horizontal line is the value 0.

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## 標準普爾信用評等是否為好的信用風險代理者？ 隨機效率前緣方法實證研究

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### 摘 要

我們使用隨機邊界模型 (stochastic frontier model; Battese and Coelli, 1995) 的公司技術無效率來評價由離散時間危險模型 (discrete-time hazard model; Shumway, 2001) 所得到的違約機率與標準普爾評等公司所公佈的長期發行者信用評等 (long-term issuer credit rating; LTR)。為了研究 LTR 在技術無效率的效用，我們使用一個特殊設計矩陣來重新表示 LTR 的秩序等級。以 1998 年到 2011 年所有美國產業公司的月資料為樣本，實證結果顯示，標準普爾評等公司所公佈的 LTR 比離散時間危險模型所得到的違約機率，顯著地提供較多公司技術無效率的資訊內容 (information content)。由於經濟效率是一個公司長期健康與前景的合理指標 (Baek and Pagán, 2002)，我們結論標準普爾評等公司所公佈的 LTR 為一個好的公司信用風險代理者。

關鍵詞：離散時間危險模型，擴大滾動視窗法，長期發行者信用評等，穩健 Wald 檢定，隨機邊界模型。

JEL classification: D20, G24.